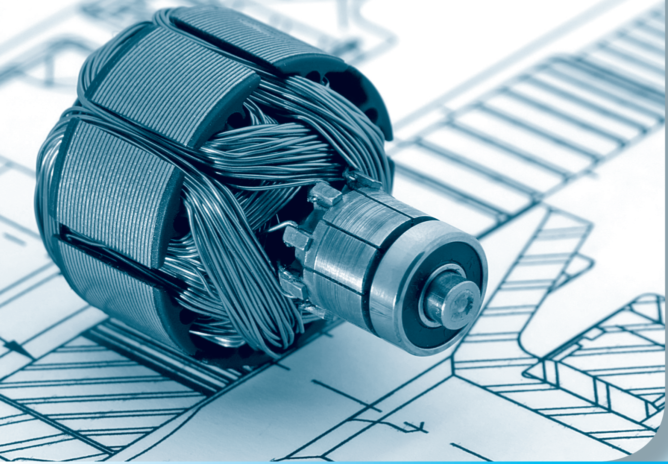

2

Transformers

What You Will Learn in This Chapter

A Theoretical Aspects

1. Principle of operation of transformers
2. 1-θ Transformers
3. Three-phase, two-winding transformers
4. Special transformers (1-θ to 3-θ and 2-θ to 3-θ)
5. Three-phase, three-winding transformers
6. Autotransformers
7. Parallel operation of transformers
8. Optical and hard-wired instrument transformers
9. Hook-up of three-phase digital meters
10. Ferroresonance
11. K-rated transformers

B Practical Highlights—Interface

1. Application of transformers
2. Measurement of the transformers' equivalent circuit parameters
3. Transformers' starting current and harmonics
4. How to draw phasor diagrams
5. Efficiency of transformers
6. Rating and one-line diagram representation (Almost all industrial transformers have a nominal rating and an increased capacity of 1.33 $\times$ nominal by just adding cooling fans at about a 10% increase in the cost of the transformer.)
7. How to solve problems involving transformers, their upstream and downstream cables, and various types of loads
8. How to solve problems of Δ-Y transformers that supply L-N, L-L, and three-phase loads
9. Why a hard-wired CT will be damaged and may cause fires when it is open-circuited while connected to an operating distribution system. Contrary to it, a PT will be destroyed when its secondary winding is shorted.
10. How the optical instrument transformers operate and how you can hook up a digital multimeter
11. Ferroresonance and preventing transformer overheating due to system harmonics
12. Governing equations
13. Manufacturers' published losses, efficiency, and impedance of transformers

C Additional Student Aid on the Web

1. Wiring diagrams for analog meters
2. Analog meters

2.0 Introduction

This chapter covers the principles of operation, equivalent circuits, losses, and applications of the various types of transformers.

Transformers, as the name implies, transform or change, from one level to another, the current and voltage that are applied to their input windings. An increase, or step-up, in the voltage across one winding is accompanied by an equal decrease, or step-down, of the current in the same winding.

Depending on the distance between the generating station and the user of electricity, the voltage—through a setup transformer—is increased, so the transmission line current is decreased and the line's energy loss and voltage drop are decreased.

Almost all power distribution within factories, homes, and elsewhere is for economical reasons of sinusoidal voltages. (It simplifies the design and operation of motors and generators and the step-up or step-down of a given voltage).

Single-phase transformers are covered in the greatest detail in this chapter because every other type of transformer is either a slightly modified single-phase transformer or a combination of single-phase transformers.

Two-winding, three-phase transformers are also discussed because they are of primary importance to industry.

Autotransformers *and* ***transformers in parallel operation*** have limited applications, but they are extensively analyzed because serious field problems may result from misunderstanding their operation.

Instrument transformers are an essential part of any distribution network. Although covered in texts on electrical measurements, they are briefly discussed here in order to point out some of the common problems that may arise during their installation and operation, and to emphasize the effects of magnetic saturation. The recently developed optical transformers are also described.

Analysis of transformers requires knowledge of ***per-unit values*** and of ***three-phase networks***, which are common to all types of ac machines. Both of these topics are briefly discussed in this chapter and are extensively covered in the Appendices of this revision of the book.

The end of this chapter includes some practical highlights of transformer applications, such as nameplate data (technical data inscribed on a metal plate fastened on an easily seen part of the transformer) and typical manufacturers' test results.

The photos of Fig. 2-1 show a large outdoor substation transformer and other transfers with small power ratings.

Fig. 2-2 is a photo of one of the 1st transformers built in North America.

The property of induction was discovered in the 1830s, but it wasn't until 1886 that William Stanley, working for Westinghouse, built the first refined, commercially used transformer. His work was built on some rudimentary designs by the Ganz Company in Hungary (ZBD Transformer 1878), and Lucien Gaulard and John Dixon Gibbs in England.

FIG. 2-1 Various types of transformers. *DSBfoto/Shutterstock.com and Courtesy of mikroElektronika*

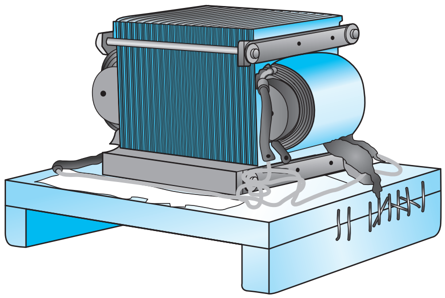

FIG. 2-2 Transformer of 1886. *Based on Stalney Transformer, http://edisontechcenter.org/LocalSites/StanleyTransformer1.jpg*

2.1 Single-Phase Transformers

As shown in Fig. 2-3, single-phase transformers usually have one input and one output winding, often referred to as the transformer's primary and secondary windings. These windings are not electrically connected but are magnetically coupled. The primary winding draws energy from a voltage source, whereas the secondary winding delivers energy to a load.

Transformers are not power amplifiers. For all practical purposes, the apparent input power ($|S|$) to a transformer's primary winding is equal to the apparent power delivered to the load by its secondary winding. In other words, the volt-amperes of the primary winding (V_1I_1) are approximately equal to the

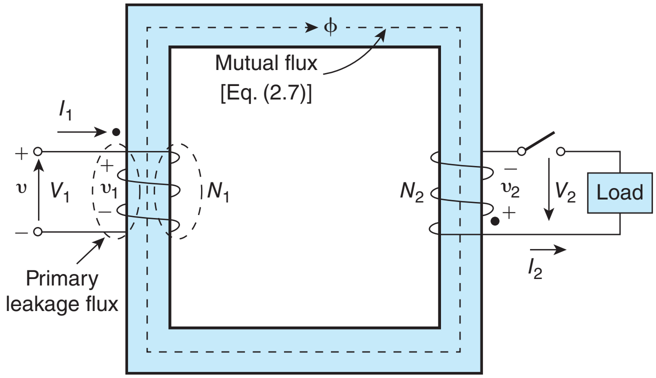

FIG. 2-3 Single-phase transformer.

volt-amperes of the secondary winding ($V_1 I_2$). Mathematically,

$$|S| = V_1 I_1 \approx V_2 I_2 \tag{2.1}$$

In actual cases—because of transformer losses—the volt-amperes delivered to a load by a transformer are slightly less than the volt-amperes drawn from the voltage source.

2.1.1 Principle of Operation

The operation of single-phase transformers (and all other transformers) is based on the principle of induction. According to this principle, a voltage is induced in a winding when the winding's flux linkages (λ) change as a function of time. The instantaneous value of the flux linkages is defined as

$$\lambda = N\phi \tag{2.2}$$

where N and ϕ are, respectively, the coil's number of turns and the instantaneous value of the flux per turn. Alternatively,

$$\lambda = Li \tag{2.3}$$

where L and i are, respectively, the coil's inductance and instantaneous current.

According to Faraday's law, the instantaneous value of the induced voltage (v) is given by

$$v = -\frac{d\lambda}{dt} \tag{2.4}$$

or

$$v = -\frac{d}{dt}(N\phi) \tag{2.5}$$

The minus sign in the equations signifies that the voltage induced opposes the supply voltage. This, however, is not necessary because the application of KVL in the corresponding loop will yield the proper sign. Furthermore, the voltage has a meaning when you identify the two terminals across which the voltage is measured. The induction principle can also be explained by using the following simple everyday logic.

The flux lines of the primary current disturb—pass through—the secondary coil that, because "action is equal and opposite to reaction," produces its own current and flux in order to oppose the disturbance. This current and/or flux is associated with the voltage developed. The voltage developed is also referred to as the voltage produced, induced, or generated.

From Eq. (2.5):

$$v = -N\frac{d\phi}{dt} \tag{2.6}$$

The negative sign in the equation signifies that the polarity of the induced voltage opposes the change that produced it. This reaction is a natural one for magnetically coupled coils and follows the principle commonly known as Lenz's law.

As shown in Fig. 2-3, the polarity of the induced voltages is usually identified by the dot symbol. The physical significance of the dot symbol is explained in detail in Chapter 1 (see Section 1.2.14).

Equation (2.6) is a basic law of electromagnetism and one of the most important mathematical relationships of electrical engineering. It governs the operation of motors and generators, and it demonstrates the coexistence of, and the quantitative relationship between, electric and magnetic fields.

The flux within the structure of the transformer changes because it is produced by the alternating voltage supplied to the transformer's input winding.

As Eq. (2.6) makes clear, for sinusoidal input voltages the flux is at a maximum when the voltage is at the zero point of its cycle. The flux and the voltage phasors must be 90° out of phase with each other. Typical voltage and flux waveforms for a transformer are shown in Fig. 2-4.

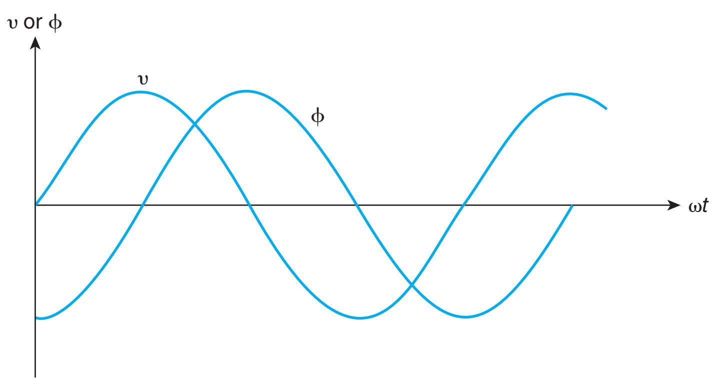

FIG. 2-4 Voltage and flux waveforms in a single-phase transformer.

The equation that relates the induced voltage to the flux produced is derived in Chapter 1 [Eq. (1.148)]. For convenience, it is rewritten here:

$$V = 4.44Nf\phi_m \quad \textbf{(2.7)}$$

where V is the rms value of the input voltage, f its frequency of oscillation in hertz, and ϕ_m the maximum value of the flux within the magnetic material in webers.

Thus, the rms value of the voltage induced in the secondary winding of the transformer, or in any other coil wound on the same core, will be given by Eq. (2.7). In each case, the appropriate number of turns must be used.

Equation (2.7) is called the fundamental transformer equation, and it is often used in laboratories to calculate the flux level within any shape—toroidal, rectangular, and so on—of magnetic circuit. This equation gives accurate results only if the leakage impedance of the coil is negligible.

The flux produced by the primary winding is divided into two parts: leakage flux and mutual flux. Leakage flux links only the windings of the primary coil and is associated with the transformer leakage impedance. Mutual flux links the windings of the primary and secondary coils and is associated with the magnetizing impedance of the transformer.

2.1.2 Ideal Transformer Relationships

In this section we derive general equations that relate the parameters of the primary winding to the parameters of the secondary winding. We will consider only ideal transformers in order to provide the basis for analysis of nonideal transformers in follow-up sections.

An ideal transformer has zero core loss, no leakage flux, and negligible winding resistances. Refer to Fig. 2-5. Using the induction principle, we have

$$v_1 = -N_1 \frac{d\phi}{dt} \quad \textbf{(2.8)}$$

$$v_2 = -N_2 \frac{d\phi}{dt} \quad \textbf{(2.9)}$$

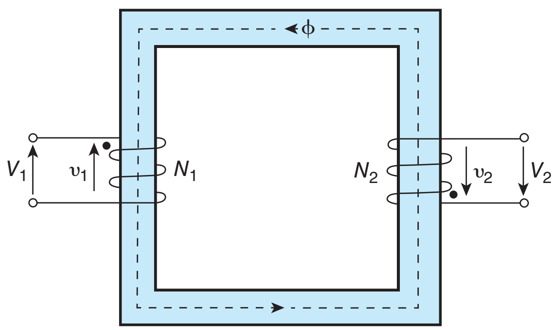

FIG. 2-5 An illustration of the concept of voltage induced. Only the mutual flux is shown.

From Eqs. (2.8) and (2.9), we obtain:

$$v_2 = v_1 \frac{N_2}{N_1} \quad \text{(2.10)}$$

Under the assumed ideal conditions, the induced voltages (v_1, v_2) are equal to their corresponding terminal voltages. Then using rms values, we obtain

$$V_2 = V_1 \frac{N_2}{N_1} \quad \text{(2.11)}$$

That is, voltages of different magnitudes can be obtained by winding coils with different numbers of turns around a magnetic circuit. The effective flux (ϕ) within the magnetic material is dependent on the applied voltage and is essentially independent of the flux of the output current. Thus, under ideal conditions, the voltage induced in the output winding is independent of the load current.

Figure 2-6 shows a schematic of a two-winding transformer, its magnetic equivalent circuit, and its ideal electrical equivalent circuit.

Figure 2-6(a) is the physical representation of an ideal transformer. From Kirchhoff's voltage law (KVL) for magnetic circuits [Fig. 2-6(b)], we have

$$\Sigma\,(NI)_{\text{loop}} = 0 \quad \text{(2.12)}$$

That is, the sum of the magnetic potentials or ampere-turns within a closed magnetic circuit is equal to zero. In mathematical symbols,

$$(N_1 I_1)_{\text{input}} - (N_c I_c)_{\text{core}} - (N_2 I_2)_{\text{output}} = 0 \quad \text{(2.13)}$$

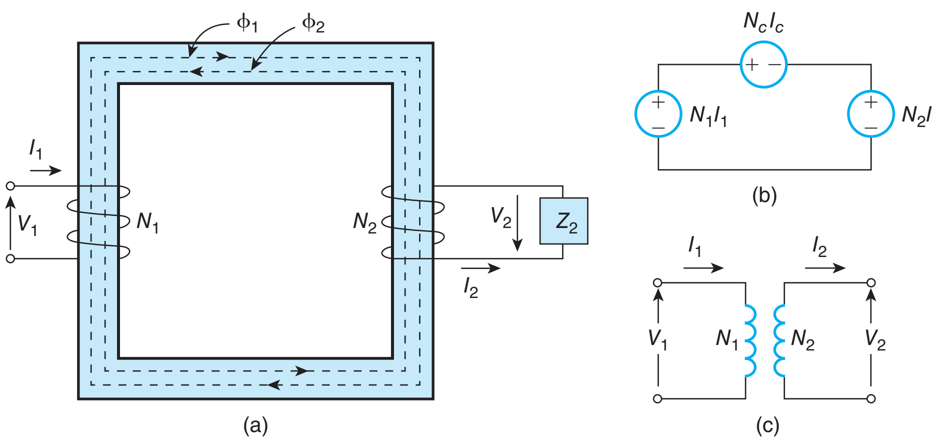

FIG. 2-6 An ideal transformer. **(a)** Physical representation. **(b)** Equivalent magnetic circuit. **(c)** Equivalent electrical circuit.

The magnetic potential $(N_c I_c)_{\text{core}}$ is the amount of magnetomotive force (mmf) required to magnetize the core of the transformer. For an ideal transformer ($\mu = \infty$), this mmf is equal to zero. That is,

$(N_c I_c)_{\text{core}} = 0.$

Thus, from Eq. (2.13), we have

$$I_2 = I_1 \frac{N_1}{N_2} \tag{2.14}$$

From Ohm's law, we obtain

$$Z_1 = \frac{V_1}{I_1} \tag{2.15}$$

and

$$Z_2 = \frac{V_2}{I_2} \tag{2.16}$$

where Z_1 and Z_2 are not the winding impedances, but the impedances as seen from the terminals of winding 1 and winding 2, respectively.

From Eqs. (2.11), (2.14), (2.15), and (2.16), we obtain the impedance transformation property of transformers. That is,

$$Z_1 = Z_2 \left(\frac{N_1}{N_2}\right)^2 \tag{2.17}$$

It should be emphasized that Eqs. (2.11), (2.14), and (2.17) are applicable only to ideal transformers. An ideal transformer is represented by the equivalent circuit shown in Fig. 2-6(c).

Example 2-1

The 5 kVA, 480–120 V, single-phase transformer shown in Fig. 2-7 delivers rated current to a 120 volt load. Neglecting losses, determine the transformer currents and the supply voltage.

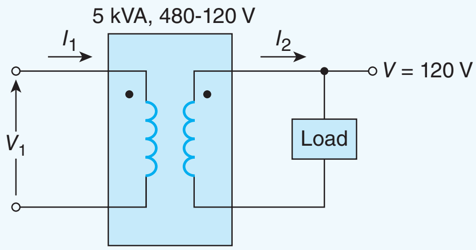

FIG. 2-7

SOLUTION

The magnitude of the current through the secondary winding is

$$I_2 = \frac{|S|}{V} = \frac{5000}{120} = \underline{\underline{41.67\text{ A}}}$$

The windings turns ratio is given by the ratio of the windings voltages:

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{120}{480} = \frac{1}{4}$$

From Eq. (2.14), the magnitude of the current through the primary winding is

$$I_1 = \frac{1}{4}(41.67) = \underline{\underline{10.42\text{ A}}}$$

The primary voltage is given by the rating of the transformer. That is,

$$\underline{\underline{V_1 = 480\text{ volts}}}$$

In an actual transformer, the primary voltage and current would be slightly higher than calculated here because of the effect of the transformer's impedances.

Exercise 2-1

A 5 kVA, 240–120 volt, single-phase transformer supplies rated current to a load at 120 volts. Determine the magnitude of the load impedance as seen from the input terminals of the transformer.

Answer 11.52 Ω

2.1.3 Derivation of the Equivalent Circuit

The analysis of transformers is greatly simplified by using a model, called the ***equivalent circuit***. Figure 2-8 shows various forms of the equivalent circuit of a single-phase transformer. Figure 2-8(a) is the schematic of a single-phase transformer, and its approximate equivalent circuit is shown in Fig. 2-8(b).

The resistance R_1 and the reactance X_1 represent the copper losses and the leakage flux of the primary winding, respectively. Similarly, the resistance R_2 and the reactance X_2 represent the copper losses and the leakage flux of the secondary winding, respectively. Then $R_1 + jX_1$ and $R_2 + jX_2$ represent the impedances

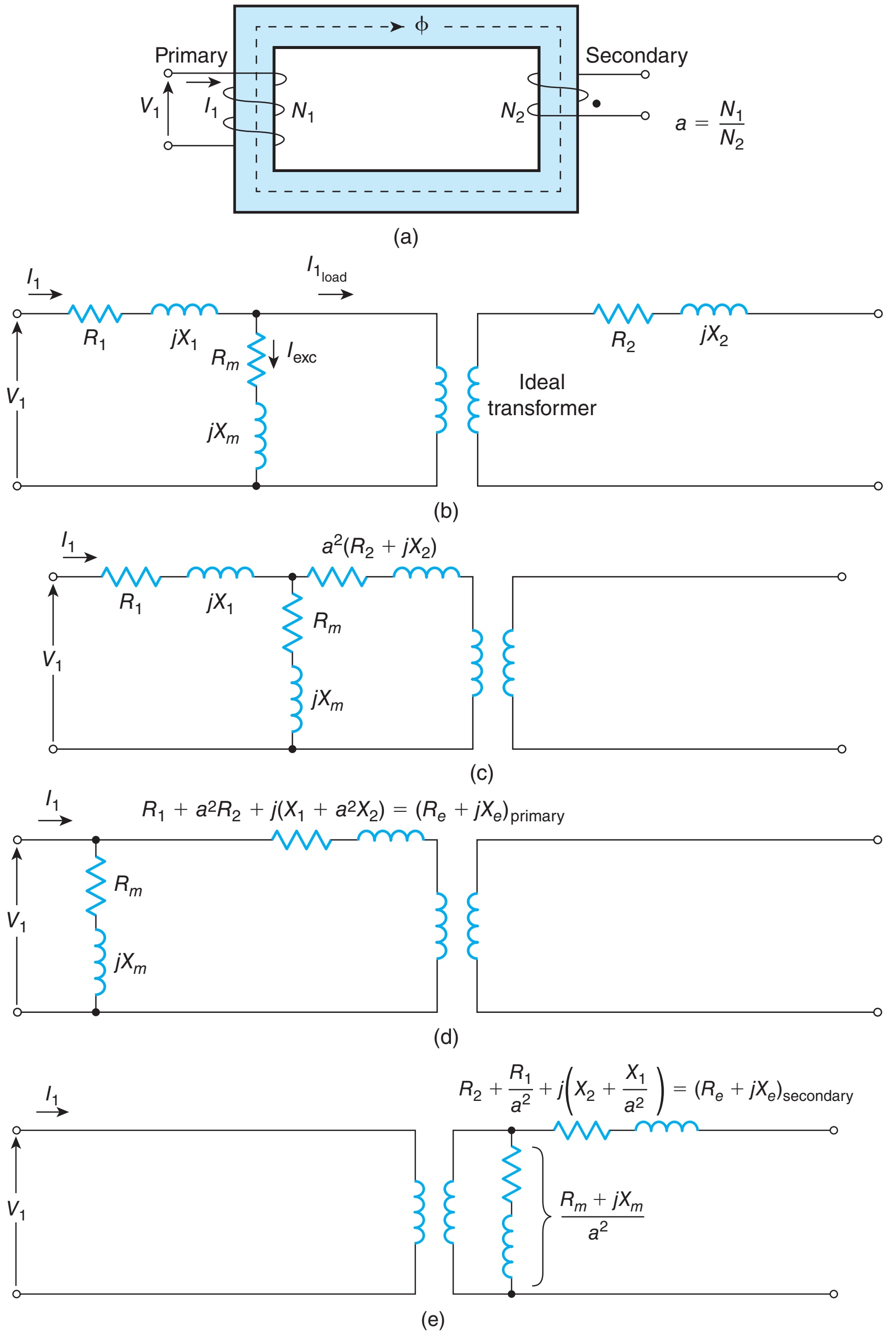

FIG. 2-8 Various forms of the equivalent circuit of a single-phase transformer. **(a)** Elementary schematic. **(b)** Individual winding impedances and the magnetizing impedance. **(c)** All impedances transferred to the primary. **(d)** An approximation of part (b). **(e)** All impedances referred or transferred to the secondary.

of the primary and secondary windings, respectively. These impedances are often referred to as leakage impedances.

The winding with the higher voltage rating has higher leakage impedance than the winding with the lower voltage rating. R_m represents the core-loss equivalent resistance, and X_m represents the magnetizing reactance of the transformer. Usually, the magnetizing impedance is represented by a core resistance (R_c) in parallel with a magnetizing reactance X_ϕ [see Fig. 2-8(c)].

The resistance R_m represents the so-called magnetizing or core losses, that is, eddy-current and hysteresis losses.

Typical core losses are given in Tables 2-6 and 2-7 (see Tables 2-6 and 2-7 in the Summary). Scientists and manufacturers constantly develop higher quality material and thus tomorrow's transformers will have lower losses.

In Fig. 2-8(c), the leakage impedance of the secondary winding is transferred to the primary.

In Fig. 2-8(d), the magnetizing impedance is relocated to the input terminals of the transformer. This equivalent circuit, though approximate, simplifies the calculations.

In Fig. 2-8(e), all impedances shown in Fig. 2-8(d) are transferred to the secondary winding.

The leakage impedance is calculated from the "short-circuit test" data, and the magnetizing impedance is calculated from the "open-circuit test" data.

Short-Circuit Test

The short-circuit test (also referred to as the impedance or copper-loss test) can be done on either side of the transformer. The input power (P_z), the applied voltage (V_z), and the input current (I_z) are measured in one winding, while the other winding, as shown in Fig. 2-9, is short-circuited. The objective of this test is to find the power loss in the windings of the transformer and the equivalent winding impedances under rated conditions. The winding power losses affect the efficiency of the transformer, and the leakage impedance affects the short-circuit current and the output voltage of the transformer. For this reason, rated current is used in this test. Thus,

$$I_z = I_{\text{rated}}$$

In practice, a near rated value is used, and then proper adjustments are made to related calculations so that the parameters obtained represent the transformer at nominal operating condition.

The voltage V_z is only a small percentage of the rated voltage and is sufficient to circulate rated current in the windings of the transformer. Usually,

$$V_z = (2\% \longrightarrow 12\%)V_{\text{rated}}$$

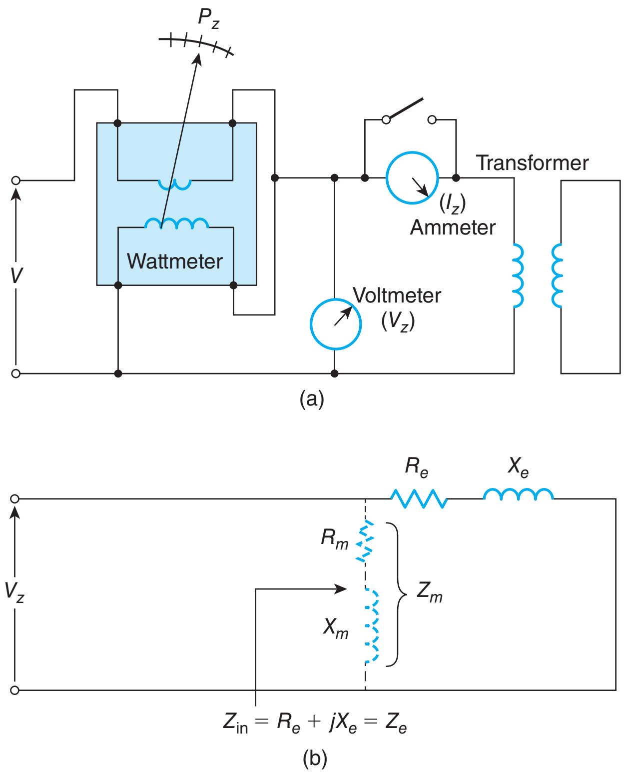

FIG. 2-9 Short-circuit test. **(a)** Laboratory connection schematic. **(b)** Equivalent circuit. (Notice that the magnetizing impedance Z_m is considered infinite.)

As a result, the transformer is magnetized at a relatively low flux density, as shown in Fig. 2-11. In performing this test, the magnetizing impedance is assumed to be open because, in relative terms, it is very large.

Neglecting the losses of the measuring instruments, we have

$$R_e = \frac{P_z}{I_z^2} \text{ ohms} \tag{2.18}$$

$$|Z_e| = \frac{V_z}{I_z} \text{ ohms} \tag{2.19}$$

and

$$X_e = \sqrt{Z_e^2 - R_e^2} \text{ ohms} \tag{2.20}$$

This test does not give the individual winding impedances, but rather the combined impedances of primary and secondary windings. The subscript *e* is used to indicate that the short-circuit test gives the equivalent transformer leakage impedance, or the combination of the winding impedances as seen from one of the transformer windings.

The winding parameters as calculated from Eqs. (2.18), (2.19), and (2.20) are said to be referred to the side of the transformer where the instruments were placed. However, you can easily transfer this impedance to the other side of the transformer by multiplying it by the turns ratio squared. The turns ratio used must be the one seen from the side of the transformer to which this impedance is to be referred.

Open-Circuit Test

The open-circuit test (also referred to as the core-loss test, the magnetization test, the excitation test, the iron-loss test, or the no-load test) furnishes the core loss and the magnetizing impedance under rated conditions. It is usually done on the side of the transformer that has the lower rated voltage.

In conducting this test, the equivalent leakage impedance of the transformer is considered negligible because it is, relatively speaking, very small. The excitation power (P_{exc}), the excitation current (I_{exc}), and the excitation voltage (V_{exc}) are measured in one winding, as shown in Fig. 2-10(a), while the other winding is open-circuited. Neglecting the losses of the measuring meters, we have

$$R_m = \frac{P_{\text{exc}}}{I_{\text{exc}}^2} \text{ ohms} \tag{2.21}$$

$$|Z_m| = \frac{V_{\text{exc}}}{I_{\text{exc}}} \text{ ohms} \tag{2.22}$$

and

$$X_m = \sqrt{Z_m^2 - R_m^2} \text{ ohms} \tag{2.23}$$

The excitation current is a small percentage of the transformer's nominal current. Usually,

$$I_{\text{exc}} = (3\% \longrightarrow 10\%)\, I_{\text{rated}}$$

In order to obtain the core loss that corresponds to rated conditions, the flux on open-circuit test must be equal to the flux within the transformer when the transformer delivers rated current. This is insured, as can be seen from Eq. (2.7), when the excitation voltage is equal to the rated voltage. That is,

$$V_{\text{exc}} = V_{\text{rated}}$$

Thus, as shown in Fig. 2-11, on an open-circuit test the transformer is energized at approximately the same flux level as under normal operating conditions.

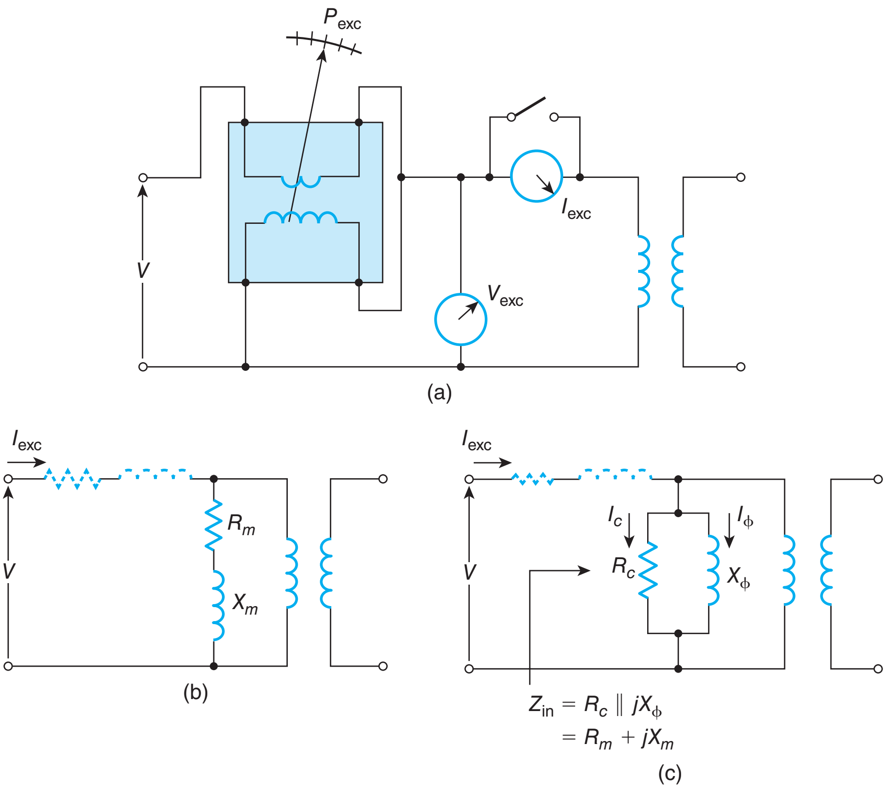

FIG. 2-10 Open-circuit test. **(a)** Laboratory connection schematic. **(b)** Series representation of the magnetizing impedance (notice that $R_1 + jX_1$ are considered negligible). **(c)** Parallel representation of the magnetizing impedance.

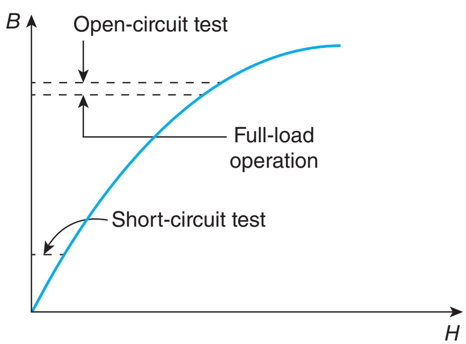

FIG. 2-11 *B-H* curve, showing the relative magnetization levels for a transformer during a short-circuit test, open-circuit test, and under full-load operating conditions.

The transformer's magnetizing parameters as calculated from Eqs. (2.21), (2.22), and (2.23) are as seen from the side of the transformer where the instruments were placed. It can be easily transferred or referred to the other side by using the impedance transformation property of ideal transformers.

When the core loss is measured at other than the operating voltage, then the actual copper losses can be calculated by considering them as being proportional to the square of the applied voltage.

In the open-circuit test, both primary and secondary voltages are customarily recorded. This gives the effective turns ratio of the transformer, which might be slightly different from the ratio specified on its nameplate.

The series representation of the magnetizing impedance can be represented by its equivalent parallel impedance, as shown in Fig. 2-10(c). The equations that relate the components of the series equivalent impedance to those of the equivalent parallel impedance, Eqs. (1.179) and (1.180), are repeated here for convenience:

$$R_c = \frac{R_m^2 + X_m^2}{R_m} \quad (2.24)$$

$$X_\phi = \frac{R_m^2 + X_m^2}{X_m} \quad (2.25)$$

The series impedance representation tends to simplify the calculations, but the parallel representation reveals more about the magnetization process and thus is more common. For example, an inspection of the parallel impedance representation gives the core losses ($P_{\text{exc}} = V^2/R_c$) and the components of the exciting current ($I_\phi = V/jX_\phi, I_c = V/R_c$). The core or open-circuit losses are caused by eddy currents and hysteresis losses. These losses are further discussed in the sections that follow. Typical winding and iron losses for a 5 kVA single-phase transformer as a function of load current are shown in Fig. 2-12.

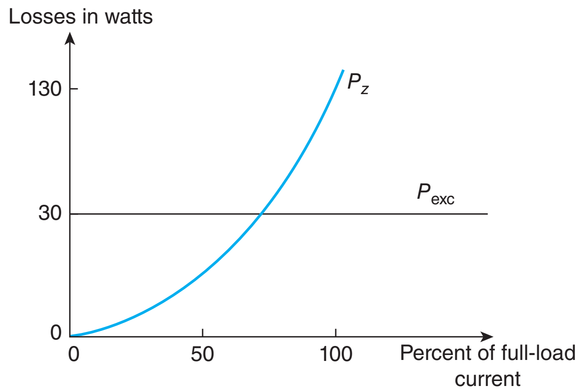

FIG. 2-12 Typical winding and iron losses as a function of load current for a single-phase transformer (5 kVA, 480–120 V).

In concluding this section on transformer tests, it must be emphasized that Z_e is the input impedance of the transformer when its secondary is shorted, and Z_m is the input impedance of the transformer when its secondary is open-circuited. These statements are, of course, approximate because of the underlying assumptions.

EXAMPLE 2-2

The following results were obtained from testing a 10 kVA, 480–120 V, single-phase transformer.

Test	*Voltage in Volts*	*Current in Amperes*	*Power in Watts*
Open-circuit	120	2.5	60
Short-circuit	26	20.83	200

Determine:

a. The equivalent leakage impedance of the transformer windings referred to the high-voltage (HV) and low-voltage (LV) winding.
b. The series and parallel components of the magnetizing branch referred to the LV winding.
c. The equivalent circuit of the transformer referred to the HV winding.

SOLUTION

a. The rated current of the transformer through the primary and secondary windings is:

$$I_H = \frac{|S|}{V} = \frac{10{,}000}{480} = 20.83 \text{ A}$$

$$I_L = \frac{10{,}000}{120} = 83.33 \text{ A}$$

By comparing the above with the given test data, it is clear that the short-circuit test was done on the HV winding.

Thus,

$$R_{eH} = \frac{P_z}{I_z^2} = \frac{200}{(20.83)^2} = 0.46\ \Omega$$

and

$$|Z_{eH}| = \frac{V_z}{I_z} = \frac{26}{20.83} = 1.25\ \Omega$$

$$X_{eH} = \sqrt{Z_{eH}^2 - R_{eH}^2} = \sqrt{(1.25)^2 - (0.46)^2}$$
$$= 1.16\ \Omega$$

Thus,

$$Z_{eH} = \underline{\underline{0.46 + j1.16\ \Omega}}$$

Using the impedance transformation property of the transformers, we obtain the equivalent impedance referred to the low-voltage side:

$$\alpha = \frac{N_2}{N_1} = \frac{120}{480}$$

Thus,

$$Z_{eL} = \left(\frac{120}{480}\right)^2 (0.46 + j1.16) = \underline{\underline{0.029 + j0.073\ \Omega}}$$

b. The equivalent series elements of the magnetizing impedance are

$$|Z_{mL}| = \frac{V_{\text{exc}}}{I_{\text{exc}}} = \frac{120}{2.5} = 48\ \Omega$$

$$R_{mL} = \frac{P_{\text{exc}}}{I_{\text{exc}}{}^2} = \frac{0.60}{(2.5)^2} = 9.6\ \Omega$$

and

$$X_{mL} = \sqrt{Z_{mH}{}^2 - R_{mH}{}^2} = \sqrt{48^2 - 9.6^2}$$

$$= \underline{\underline{47.03\ \Omega}}$$

The parallel components of the magnetizing impedance, as seen from the *LV* winding, are calculated from the series equivalent components by using Eqs. (2.24) and (2.25):

$$R_{cL} = \frac{R_m^2 + X_m^2}{R_m} = \frac{9.6^2 + 47.03^2}{9.6} = \underline{\underline{240\ \Omega}}$$

and

$$X_{\phi L} = \frac{R_m^2 + X_m^2}{X_m} = \frac{9.6^2 + 47.03^2}{47.03} = \underline{\underline{48.99\ \Omega}}$$

c. Referring to the resistance and the reactance of the magnetizing branch to the HV winding, we obtain

$$R_{cH} + jX_{\phi H} = \left(\frac{480}{120}\right)^2 (240 + j48.99)$$

$$= (3840 + j783.84)\ \Omega$$

The equivalent circuit referred to the HV winding is shown in Fig. 2-13.

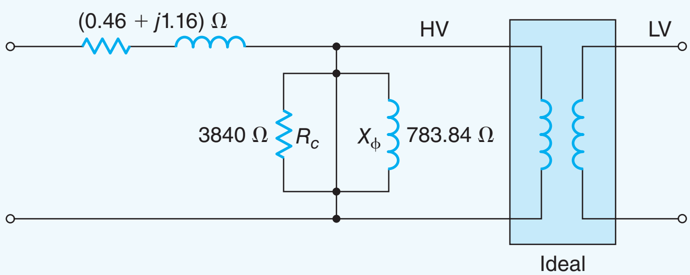

FIG. 2-13

2.1.4 Waveform of Excitation Current

When one of the transformer's windings is open-circuited while the other is connected to rated voltage, the resulting current represents a small percentage of the transformer's rated current. This is due to the transformer's high magnetization impedance and to the infinite impedance seen by one winding while the other is open-circuited. This current is referred to as the excitation current (I_{exc}). In practice,

$$I_{\text{exc}} = (3\% \rightarrow 10\%)\, I_{\text{rated}} \tag{2.26}$$

The theoretical derivation of the exciting current's waveform is obtained from the transformer's B-H characteristic as follows: Draw the waveform of the sinusoidal flux on an ωt-axis. Then, on the same coordinate system, draw the waveform of the exciting current. Refer to Fig. 2-14(a). The waveform of the flux can easily be obtained, because, it will be noted, the points $(\alpha_1, 0)$, (d_1, d_2), and $(g_1, 0)$ are $\pi/2$ radians apart and correspond, respectively, to zero, maximum, and zero values of the flux. In other words, the critical points of the flux are known during half the period of its sinusoidal function. The resulting flux waveform is sketched in Fig. 2-14(b).

The magnitude of the exciting current at each of the coordinate points [$(\alpha_1, 0)$, (b_1, b_2), (d_1, d_2), etc.] of the B-H curve is given by the abscissa of these points. This is shown in Fig. 2-14(b). The transformer's voltage, flux, and exciting current are shown in Fig. 2-14(c).

The exciting current is not sinusoidal because of the nonlinearities of the B-H curve, and so a conventional phasor diagram of the exciting current cannot be drawn. However, its harmonics content, or its equivalent sinusoidal functions, can be found by using the Fourier series analysis. The Fourier series method is a mathematical tool used to determine the harmonic content of nonsinusoidal waveforms.

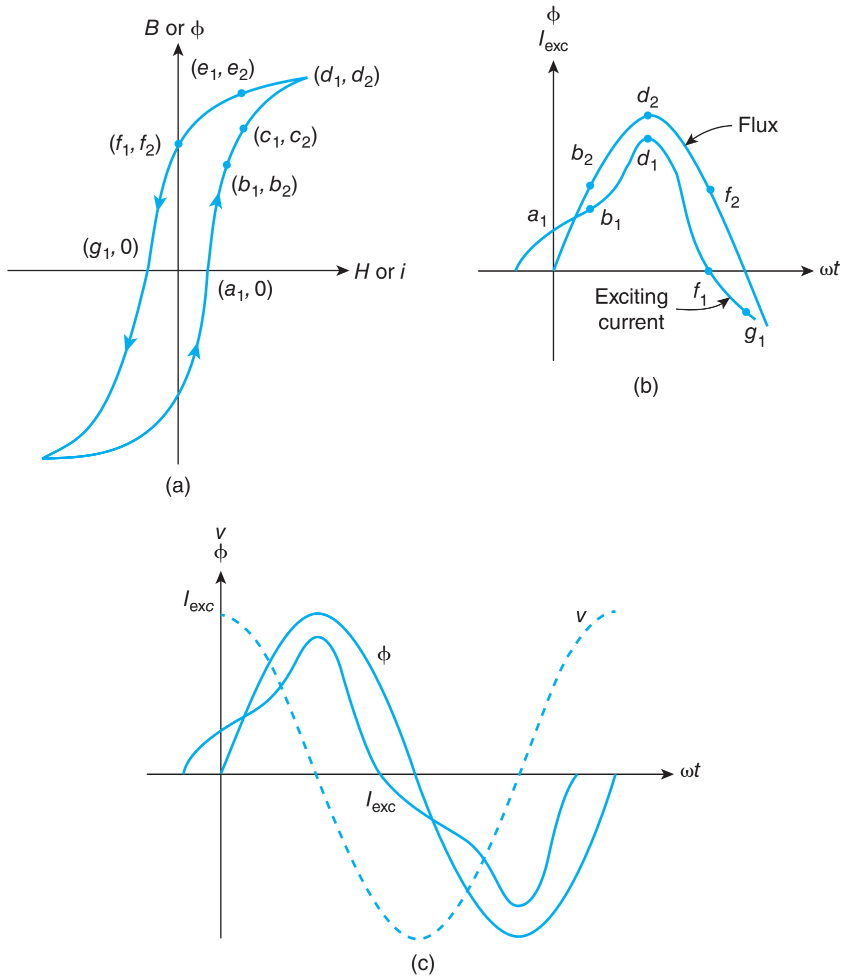

FIG. 2-14 Typical magnetic characteristics of transformers. **(a)** *B-H* curve. **(b)** Instantaneous values of flux and exciting current. **(c)** Instantaneous values of induced voltage, flux, and exciting current.

It can be shown that the exciting current is made up of "odd harmonics,"* that is, a fundamental term, a third harmonic, a fifth harmonic, and so on. The third harmonic can be as high as 40% of the fundamental term.

The exciting current is very important in the operation of all transformers, particularly three-phase transformers. It is discussed in greater detail in Section 2.2.4.

*According to Fourier series analysis, any function $f(t)$ of period T has only *odd* harmonics if $f(t)$ satisfies the following relationship: $f(t) = -f[t + T/2]$. The function that has such a characteristic is said to have half-wave *odd* symmetry.

2.1.5 Components of Primary Current and Corresponding Fluxes

When a transformer is connected to its primary supply voltage while its secondary is open-circuited, it draws (as stated previously) a small percentage of its rated current (I_{exc}). Although this current is small, it produces rated flux within the core of the transformer.

As soon as an impedance is connected to the transformer's secondary (see Fig. 2-15(a)), a load current will flow. This current will produce the flux (ϕ_2) that opposes the flux of the primary current. This cannot be tolerated because it would be accompanied by a reduction of the voltage induced in the primary [$v_i = -N(d\phi/dt)$], which would result in violation of KVL ($V_1 = v_i$). To overcome this opposition, the current drawn by the primary winding *increases* in such a way as to completely cancel the magnetic opposition of the secondary current.

The quantity of primary current needed to produce flux sufficient to completely neutralize the opposition of the secondary current is called the load component of the primary current. It is designated by I_{1L}. Alternatively, the mmf of the secondary winding (N_2I_2) is equal and opposite to the increase in the mmf of the primary winding (N_1I_{1L}). As a result, the effective mmf of the transformer at negligible leakage flux is N_1I_{exc}.

The increase in the primary current that accompanies the flow of current in the secondary could also be justified by considering the following principle of energy conservation:

$$\begin{pmatrix}\text{energy drawn by}\\ \text{the transformer}\end{pmatrix} = \begin{pmatrix}\text{energy delivered}\\ \text{to the load}\end{pmatrix} + \begin{pmatrix}\text{energy consumed within}\\ \text{the transformer}\end{pmatrix}$$

The energy demand of the load is met by an increase in the magnitude of the primary current and its power factor relative to the no-load condition.

From the previous discussion it is evident that, under nominal operating conditions, the primary current is made up of two components. One, the exciting current, is required to magnetize the transformer; the other, its load component (L_{1L}), is needed to cancel the opposition of the secondary current. In mathematical form, and as shown in Fig. 2-15(b),

$$I_1 = I_{exc} + I_{1L} \tag{2.27}$$

The primary current I_1 is almost sinusoidal because its nonsinusoidal component, the exciting current, is negligible in comparison to its sinusoidal load component.

The flux of the primary current is shown in Fig. 2-15(a). Its mathematical representation is

$$\phi_1 = \phi_{exc} + \phi_{1L} \tag{2.28}$$

where ϕ_{1L} is the load component of the primary flux.

Phasor Diagram

Refer to Fig. 2-15(b). The transformer delivers power to a lagging-power-factor load. The windings turns ratio and the load voltage V_L normally are known. The load current I_2 is obtained from the load characteristics. The basic circuit equations are

$$V_1 N_2 = V_{nl} N_1 \tag{2.29}$$

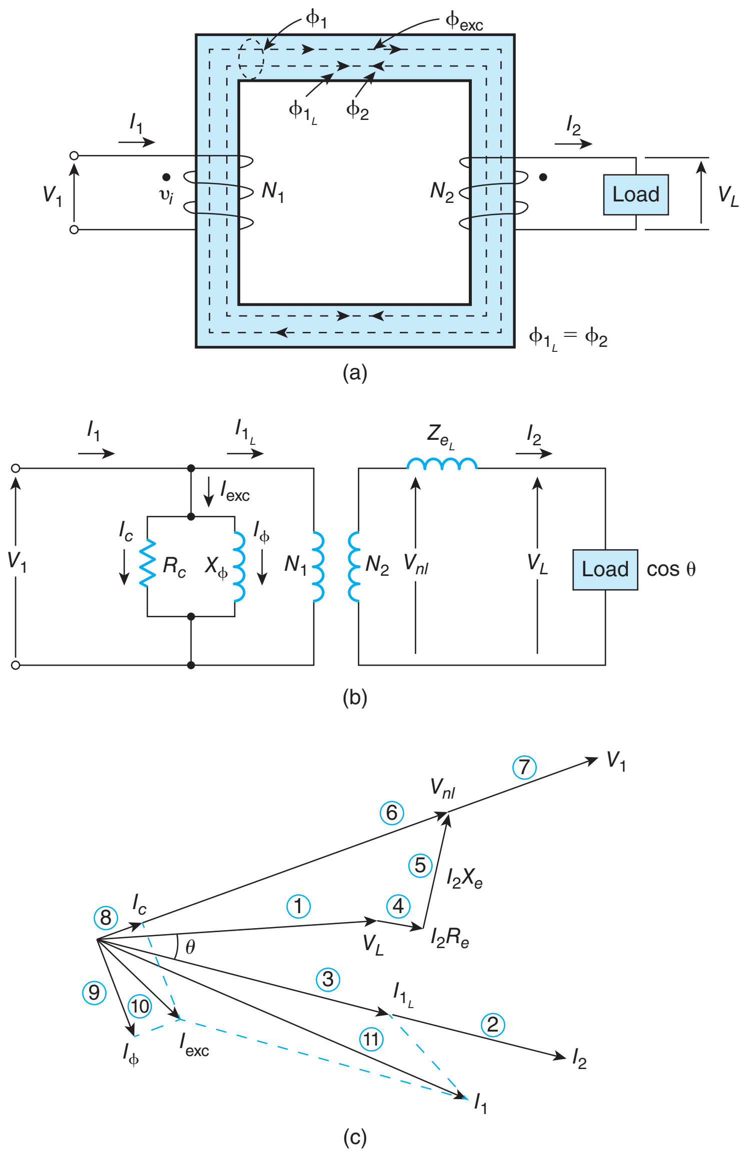

FIG. 2-15 Transformer. **(a)** Polarity and component fluxes. **(b)** Equivalent circuit. **(c)** Phasor diagram.

and

$$V_{nl} = V_L + I_2 Z_{eL} \tag{2.30}$$

These equations, together with Eq. (2.27), are used to construct the phasor diagram shown in Fig. 2-15(c). The encircled numbers indicate the sequence of steps that you may follow in order to simplify its construction.

2.1.6 Transformer Characteristics

Inrush Current

When switching ON a transformer, the input or the inrush current is often many times larger than its rated current. This occurs when the switching takes place at the instant the input voltage is at the zero point on its time cycle. This results in a maximum flux that corresponds to a near saturation point on the *B-H* curve. At this point, the permeability of the magnetic material, the corresponding inductance, and the magnetizing impedance are at a minimum. Thus, the resulting current is at a maximum.

The magnitude and the waveform of the inrush current also depend on the leakage impedance of the transformer, on its magnetization characteristic, on the residual flux, and on the magnitude of the applied voltage at the instant of switching. The current waveform is such as to satisfy KVL. It cannot be accurately described because of the variable and nonlinear circuit characteristics. You can reduce the magnitude of the inrush current by selecting the transformer's protective circuit breaker with a microprocessor. The microprocessor switches ON the breaker when the supply voltage is zero.

The magnitude of the inrush current is comparable to the current that results from external transformer shorts. However, the short-circuit current is of the

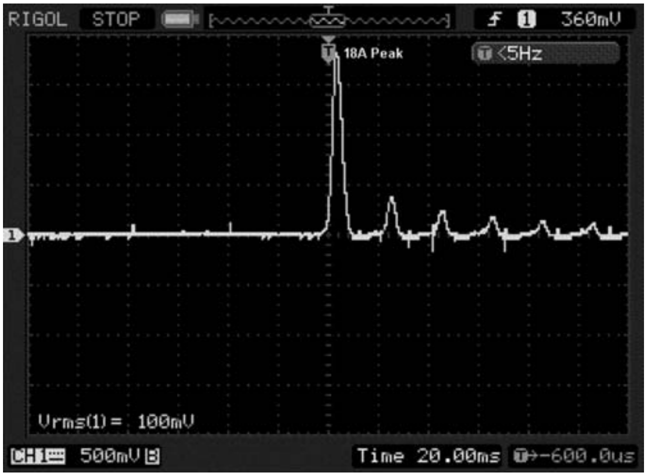

FIG. 2-16 The inrush waveform that is captured when power is applied at the main zero crossing point. *Courtesy of ESP*

fundamental frequency, while the magnetizing current has a large content (up to 40%) of the third harmonic. The magnitude of the inrush current, as per standard industrial practice, is taken as being between 8 and 12 times the rated current of the transformer, and its duration is about 100 milliseconds. As such, it is important in the selection of the upstream protective devices.

Efficiency

An unloaded transformer when connected to its voltage source, draws only the magnetization current on the primary side, the secondary current being zero. As the load is increased, the primary and secondary currents increase as per the load requirements. The volt amperes and wattage handled by the transformer also increase. Due to the presence of no load losses and I^2R losses in the windings, a certain amount of electrical energy gets dissipated as heat inside the transformer. This gives rise to the concept of efficiency (http://nptel.iitm.ac.in/courses/IIT-MADRAS/Electrical_Machines_I/pdfs/1_10.pdf).

By definition, the efficiency (η) of a transformer is given by:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{in}} - P_{\text{loss}}}{P_{\text{in}}} = 1 - \frac{P_{\text{loss}}}{P_{\text{in}}} \tag{2.31}$$

where P_{in} and P_{out} correspond, respectively, to the input and output power of the transformer expressed in watts. The power loss (P_{loss}) is the sum of the core and copper losses of the transformer at the operating conditions under consideration. The copper loss depends on the kVA delivered. When the kVA drawn by the load varies over a 24-hour period, then the so-called 24-hour transformer efficiency may be calculated by using energy instead of power. Generally speaking, the efficiency of transformers increases with their rating.

Industry continuously tries to minimize core loss by developing higher-quality magnetic materials from which the transformers can be economically manufactured. Where economics permit, copper windings are used instead of aluminum because the aluminum gives higher winding loss (I^2R).* The efficiency of the transformer is of primary importance because the cost of energy loss within the transformer over its lifetime is usually greater than that of the transformer itself. Typical transformer losses are shown in Tables 2-5 through Table 2-8 (see Summary). In the absence of winding condensation, idle transformers should be disconnected from their supply lines; otherwise, their energy consumption, due to core loss, is wasted.

Refer to Fig. 2-17. From basic definitions, we have

$$\text{efficiency} = \frac{\text{output power}}{\text{output power} + \text{losses}} \tag{2.32}$$

or

$$\eta = \frac{V_L I_L \cos\theta}{V_L I_L \cos\theta + P_{\text{exc}} + R_e I_L^2} \tag{2.33}$$

* The resistance of an aluminum conductor with the same current capacity as that of a copper conductor is about 10% higher.

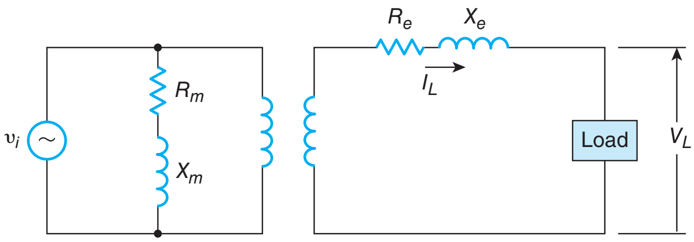

FIG. 2-17 Transformer's equivalent circuit and secondary load.

and

$$\eta = \frac{KI_L}{KI_L + P_{\text{exc}} + R_e I_L^2} \tag{2.34}$$

where

$$K = V_L \cos\theta \tag{2.35}$$

Taking the derivative of the efficiency with respect to the variable parameter I_L, we obtain

$$\frac{d\eta}{dI_L} = \frac{(KI_L + P_{\text{exc}} + R_e I_L^2)K - KI_L\,(K + 0 + 2R_e I_L)}{(KI_L + P_{\text{exc}} + R_e I_L^2)^2} \tag{2.36}$$

Setting the last equation equal to zero gives

$$P_{\text{exc}} = R_e I_L^2 \tag{2.37}$$

or

$$P_{\text{exc}} = P_z \tag{2.38}$$

Thus, the efficiency of a transformer at a constant load power factor is maximum when the iron loss is equal to its winding loss.

A transformer's efficiency depends on the load current and its power factor. The first condition is obvious, while the second becomes evident when you consider that the primary current, being the vector sum of the exciting current and the component of the load current (see Fig. 2-15(b)), depends on the power factor of the load. The magnitude of the primary current, in turn, controls the primary winding's copper losses. When you speak of a transformer's efficiency, then you must always identify the corresponding current and the power factor.

The variation of efficiency versus load current, as a function of the load current's power factor, is shown in Fig. 2-18.

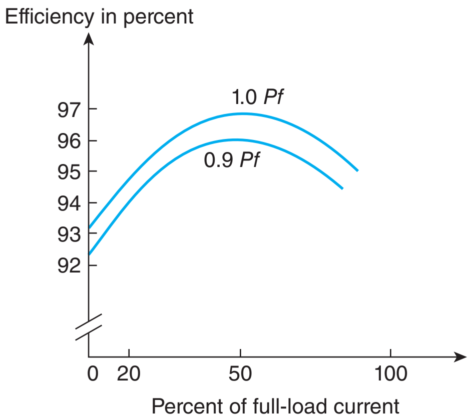

FIG. 2-18 Efficiency as a function of load current and power factor.

The variation of iron loss, winding loss, and efficiency versus load current for a 25 kVA transformer is shown in Fig. 2-19.

Regulation

Voltage regulation is a measure of the change in the magnitude of the output voltage while the load current varies from zero up to its rated value. In mathematical form, regulation is given by

$$\text{regulation } \% = \frac{|V_{nl}| - |V_{fl}|}{|V_{fl}|}(100) \qquad \textbf{(2.39)}$$

where V_{nl} and V_{fl} are, respectively, the magnitudes of the voltages at the output terminals of the transformer at no load and at full load. Full-load voltage is also

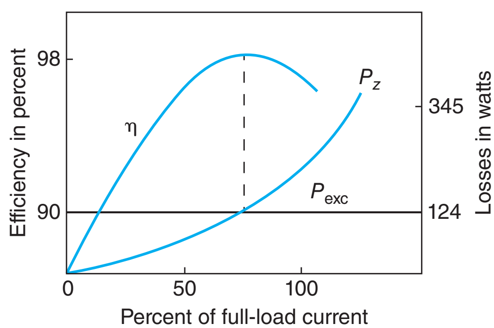

FIG. 2-19 Efficiency, iron loss, and winding loss as a function of load current for a 25 kVA, 480-120 V, single-phase transformer.

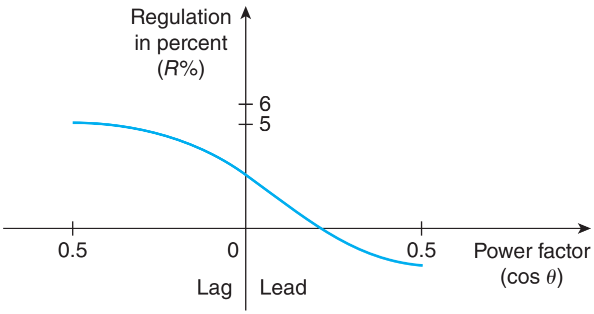

FIG. 2-20 Regulation as a function of load power factor for constant rated current.

referred to as nameplate voltage, rated voltage, or nominal transformer voltage. No-load voltage can be obtained from the input voltage and the nominal voltage ratio of the transformer. Equation (2.39) can be used for any operating condition. In such cases, the full-load voltage is replaced by its actual value.

Regulation depends on the leakage impedance of the transformer and on the power factor of the load. At a particular value of load power factor and leakage impedance, voltage regulation becomes ideal or zero. Depending on the power factor of the load, the regulation could be positive or negative. Usually, under steady-state operating conditions, it is less than plus or minus 5% ($\pm 5\%$). Regulation as a function of power factor is shown in Fig. 2-20. The effects of the load's power factor and the transformer's leakage impedance are demonstrated in Example 2-4.

At leading power factors, regulation is usually negative; that is, the voltage at the secondary terminals of a transformer is larger at full load than it is when the load is disconnected. In such cases, the equipment connected to a transformer's secondary may be subjected to higher than rated voltages. This may occur when the power-factor-correction capacitor banks remain on the network while the plant operates at a reduced load.

The voltage across the load is of primary importance because many operating characteristics of various pieces of equipment depend on it. For example, a 5% reduction of the rated voltage of an incandescent lamp noticeably reduces its output, and the light appears dimmer. Furthermore, the torque delivered by the motors, as explained in Chapter 3, is reduced by a factor of 10% ($T \alpha V^2$). For these reasons, most power distribution transformers, as per manufacturers' standards, are equipped with "off-load" voltage tap changers through which the voltage can be changed by $\pm 2.5\%$, or by $\pm 5\%$ in relation to their nominal voltage. When the voltage is to be changed, the load of the transformer is disconnected and the number of turns of the primary winding is changed according to the requirements.

kVA Rating

The apparent power, or the kVA rating, of a transformer is always inscribed on its nameplate. It indicates the transformer's transformation capacity in terms of the volt-amperes a transformer is designed to deliver.

The capability of a transformer is limited by heating in the windings (hence, there is a maximum permissible value of sustained current) and by excessively high exciting current and excessive core loss (hence, there is a maximum permissible value for sustained voltage). Rated volt-amperes is the product of rated voltage and nominal current. In normal operation, the input voltage is close to the rated value.

As long as a transformer delivers rated or reduced kVA, it will operate without being overheated. However, when it is cooled, it can dissipate more than nominal heat, and, as a result, it can safely deliver higher-than-rated kVA. For this reason, most substation transformers are equipped with a set of ventilating fans, which enable the transformer's kVA capacity to be increased proportionally to the ventilation furnished.

A transformer of 1000 kVA supplied with fans, for example, can deliver 1333 kVA without being overheated. Such a transformer is identified as 1000/1333 kVA. Transformers with higher kVA ratings can transform higher magnitudes of voltage and current; that is, their windings have, on a relative basis, larger diameters and more insulation. As a result, the *physical* size and the *cost* of a transformer depend on its kVA rating.

The importance of this parameter is further explained in Section 2.6, which provides information about all the important parameters that appear on the nameplate of a transformer. A familiarity with nameplate data facilitates the solution of practical and theoretical transformer problems.

2.1.7 Per-Unit Values

When engineering parameters such as power, torque, speed, voltage, current, and impedance are expressed in their corresponding per-unit values, many engineering concepts and calculations are simplified. For this reason, almost all design and problem solving in the power-distribution field is implemented by expressing all pertinent parameters in their equivalent per-unit values.

Actual engineering parameters are changed to their equivalent per-unit values as follows. The per-unit value of a parameter K is equal to its actual value divided by the base value. That is,

$$K = \frac{\text{actual value of } K}{\text{base value of } K} \text{ pu} \tag{2.40}$$

Base parameters are normally obtained from the nameplate data of the transformer, as follows:

$$\text{Base power } (S_b) = \text{volt-ampere rating of the transformer} \tag{2.40a}$$

$$\text{Base voltage } (V_b) = \text{rated voltage of the winding under consideration} \tag{2.40b}$$

$$\text{Base current } (I_b) = \text{rated current of the winding under consideration}$$

$$= \frac{S_b}{V_b} \tag{2.40c}$$

$$\text{Base impedance } (Z_b) = \frac{\text{base value of voltage}}{\text{base value of current}} \tag{2.40d}$$

From Eqs. (2.40c) and (2.40d), we obtain

$$Z_b = \frac{V_b^2}{S_b} \tag{2.41}$$

The base power is the same in both the high- and the low-voltage windings ($S_{bH} = S_{bL}$). In contrast, the base voltage, base current, and base impedance are different for each winding of the transformer, and thus the subscript H or L should be included in the above expressions to represent parameters at the high- and low-voltage windings. The base value is the value of the parameter expressed in standard engineering units.

When the ohmic value of an impedance is $15\angle 40°$ ohms and the value of the base impedance is 300 ohms, then the per-unit value of the impedance is

$$Z = \frac{15\angle 40°}{300} = 0.05\angle 40° \text{ pu}$$

Similarly, when the actual value of a voltage is 250 volts, and the base voltage is 200 volts, then the per-unit value of the voltage is

$$V = \frac{250}{200} = 1.25 \text{ pu}$$

The per-unit value of a parameter, multiplied by 100, gives the value of the parameter in percent. Thus, when the per-unit value of a parameter is 0.04, its percentage value is 4%.

One advantage of the per-unit system is that when the voltage, current, and impedance of a transformer are expressed in per-unit , they have the same values regardless of the side of the transformer to which they refer. This greatly simplifies the understanding and solution of various transformer problems.

An additional advantage is that when the parameters of a machine are expressed as per-unit, they all fall within a known range, regardless of the rating of the machine. The per-unit values of the equivalent leakage impedance of most single-phase transformers, for example, are within the range of 0.035–0.055 pu. Knowing this is of tremendous importance, greatly simplifying the solution of problems and the conceptual understanding of various transformer operating conditions.

It can be shown that the per-unit value of the transformer's leakage impedance is equal to the voltage required on a short-circuit test, expressed as per-unit of the rated value. That is,

$$Z_{e_{pu}} = V_{z_{pu}} \tag{2.42}$$

This is left as a student problem (see Exercise 2-3).

Because the per-unit system is used in most of the chapters of this text, it is described in detail in the Appendix.

One-Line Diagrams

Transformers, cables, and electric machines are conventionally identified in electrical drawings by their equivalent one-line representations. As the name implies, one-line diagrams represent electrical loads as being supplied through one wire, regardless of how many wires are used in the actual setup.

The main parameters of a system—per-unit impedances, kVA rating of transformers, kW rating of motors, size of cables, protective devices, the short-circuit MVA of the supply voltage source, and so on—are written on one-line diagrams.

For a single-phase system, the short-circuit apparent power ($|S|$) in volt-amperes is

$$|S| = VI_{sc} \tag{2.43}$$

where V and I_{sc} are, respectively, the nominal voltage and the short-circuit current. When the voltage is expressed in kV and the short-circuit current in kA, then Eq. (2.43) gives the short circuit apparent power in MVA.

From Eq. (2.43) and Ohm's law, we obtain

$$|S| = V\frac{V}{Z} \tag{2.44}$$

where Z is the impedance of the supply network in ohms/phase. From Eqs. (2.43) and (2.44), we obtain

$$Z = \frac{V^2}{|S|} \tag{2.45}$$

For a three-phase system, the short-circuit MVA is

$$S = \sqrt{3}\, V_{L\text{-}L} I_{sc} \tag{2.46}$$

where $V_{L\text{-}L}$ and I_{sc} are, respectively, the line-to-line nominal voltage expressed in kV and the short-circuit current expressed in kA. Thus, the short-circuit MVA of a given voltage source can be used to find the internal impedance of the source and/or the short-circuit current.

Protective equipment, such as circuit breakers and fuses, must be capable of safely withstanding the forces produced by the short-circuit currents. Since the short-circuit currents are given indirectly in terms of short-circuit MVA, it is customary in the industry to rate the short-circuit capacity of equipment in terms of short-circuit MVA.

Typical short-circuit MVA's for a 25 kV and 46 kV distribution system are as follows:

kV	MVA
25	500
46	1500

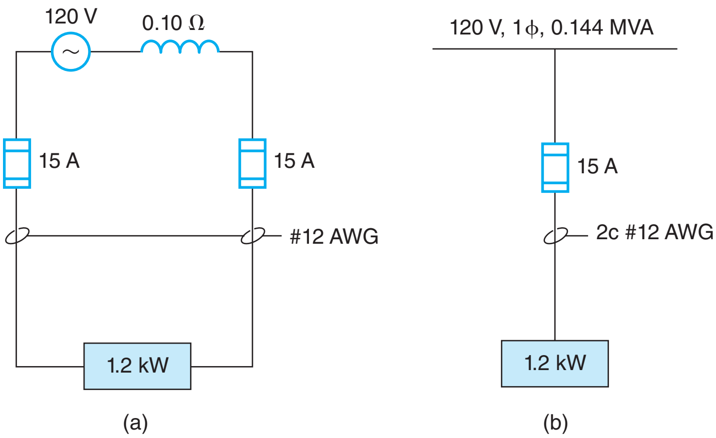

FIG. 2-21 Representation of a distribution system. **(a)** Two-wire. **(b)** One-line.

Under ideal conditions, the source impedance is equal to zero and its frequency is constant. Such a system is identified on the one-line diagram as an "infinite bus." In more general terms an infinite bus is a power source whose voltage and frequency remain constant independent of the active or reactive power being supplied.

Since all electrical drawings represent electrical loads by their one-line diagram, an attempt is made throughout this book to use the one-line diagram as often as possible. Figure 2-21(a) shows a two-wire diagram of a single-phase load. The 120 V, 60 Hz voltage supply has an internal reactance of 0.1 ohm. The 15 A fuses protect the 1.2 kW load from undesirable high currents. The two-conductor (2c) cable is size #12 AWG* (American Wire Gauge). When the load is shorted, the resulting short-circuit MVA is

$$|S| = V\,|I|$$

$$= 120\left(\frac{120}{0.1}\right) \times 10^{-6}$$

$$= 0.144 \text{ MVA}$$

The one-line diagram of Fig. 2-21(a) is shown in Fig. 2-21(b).

EXAMPLE 2-3

A 5 kVA, 480–120 V single-phase transformer delivers rated current at 0.9 pf lagging. The transformer's equivalent leakage impedance referred to the high-voltage winding is $0.92 + j1.84$ ohms. Find the per-unit values of the voltage, current, and leakage impedance on the HV and LV winding.

*3.31 mm^2

SOLUTION

The approximate equivalent circuit of the transformer is shown in Fig. 2-22.

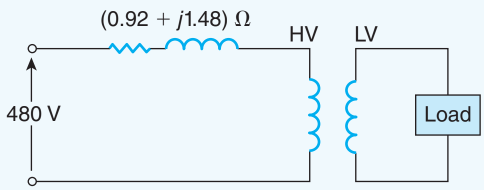

FIG. 2-22

HV Side

The base parameters are

$$S_b = 5000 \text{ VA}$$

$$V_{bH} = 480 \text{ V}$$

$$I_{bH} = \frac{5000}{480} = 10.42 \text{ A}$$

$$Z_{bH} = \frac{(480)^2}{5000} = 46.08 \text{ ohms}$$

The per-unit values are

$$V_H = \frac{\text{actual value}}{\text{base value}} = \frac{480}{480} = \underline{\underline{1.0 \text{ pu}}}$$

$$I_H = \frac{10.42\angle -25.8^\circ}{10.42} = \underline{\underline{1.0\angle -25.8^\circ \text{ pu}}}$$

$$Z_H = \frac{0.92 + j1.84}{46.08} = \underline{\underline{0.02 + j0.04 \text{ pu}}}$$

LV Side

The base parameters are

$$S_b = 5000 \text{ VA}$$

$$V_{bL} = 120 \text{ V}$$

$$I_{bL} = \frac{5000}{120} = 41.67 \text{ A}$$

$$Z_{bL} = \frac{(120)^2}{5000} = 2.88 \text{ ohms}$$

The per-unit values are

$$V_L = \frac{120}{120} = \underline{\underline{1.0 \text{ pu}}}$$

$$I_L = \frac{41.67\angle -25.8^\circ}{41.67} = \underline{\underline{1.0\angle -25.8^\circ \text{ pu}}}$$

The leakage impedance referred to the LV side is

$$Z_e = (0.92 + j1.84)\left(\frac{120}{480}\right)^2 = 0.06 + j0.12\ \Omega$$

$$= \frac{0.06 + j0.12}{2.88} = \underline{\underline{0.02 + j0.04 \text{ pu}}}$$

EXAMPLE 2-4

The 100 kVA, 440–220 V, 60 Hz single-phase transformer shown in the one-line diagram representation (Fig. 2-23(a)) has negligible magnetizing current and a leakage impedance of 0.03 + j0.040 pu. The transformer is connected to a 440 V source through a feeder whose impedance is 0.04 + j0.08 ohms.

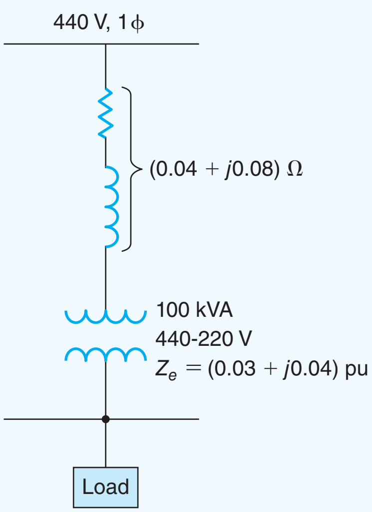

FIG. 2-23(a)

Determine:

a. The voltage across the load when the transformer is connected to a 440 V source and delivers rated current to a load of 0.80 power factor lagging.

b. The voltage regulation.

c. The primary and secondary short-circuit MVA when the secondary of the transformer is shorted.

d. The rating of the capacitor in kVAR which, when connected across the load, will improve the power factor from 0.80 lagging to unity.

SOLUTION

a. The base impedance in the HV winding is

$$Z_{bH} = \frac{(440)^2}{100{,}000} = 1.94\ \Omega$$

The per-unit value of the voltage is

$$V_{\text{pu}} = \frac{\text{actual value}}{\text{base value}} = \frac{440}{440} = 1.0 \text{ pu}$$

Similarly,

$$I_{\text{pu}} = 1.0\angle{-36.9°} \text{ pu}$$

The per-unit value of the feeder impedance is

$$Z_f = \frac{\text{actual value}}{\text{base value}}$$

$$Z_f = \frac{0.04 + j0.08}{1.94} = 0.02 + j0.04 \text{ pu}$$

Applying KVL to the equivalent circuit of Fig. 2-23(b) and using per-unit values, we have

$$V_L = V_1 - IZ$$

or

$$V_L\angle{0°} = 1.0\angle{\beta_1°} - 1.0\angle{-36.9°}\,[0.03 + 0.02 + j(0.04 + 0.04)] \qquad \textbf{(I)}$$

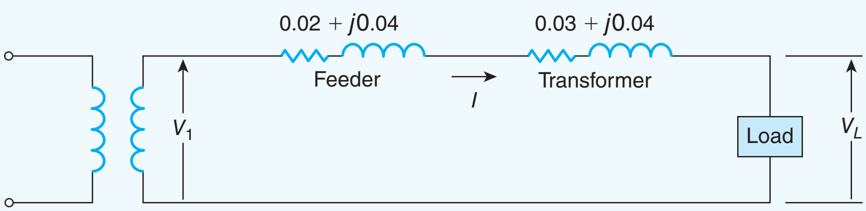

FIG. 2-23(b)

The given power factor of the load establishes the reference for the phase angles.

In other words, the voltage across the load has a phase angle equal to zero, while the input voltage will be at an angle of β_1 degrees to the reference.

From Eq. (I), and after some algebraic calculations, we obtain

$$V_L\underline{/0^\circ} = 1.0\underline{/\beta_1^\circ} - 0.096\underline{/21.2^\circ}$$

Equating the imaginary components of the above equation, we get

$$\beta_1 = \arcsin\,(0.096 \sin 21.2^\circ) = 2^\circ$$

Equating the real components, we obtain

$$\begin{aligned} V_L &= 1.0 \cos 2^\circ - 0.096 \cos 21.2^\circ \\ &= 0.91 \text{ pu} \\ &= 0.91\,(220) \\ &= \underline{\underline{200.21 \text{ V}}} \end{aligned}$$

Alternatively, using *ohmic values*, from KVL we have

$$Z_t = 1.94\,(0.03 + j0.044) = 0.06 + j0.08\ \Omega$$

The total impedance referred to the HV winding is

$$Z = (0.06 + j0.08) + (0.04 + j0.08) = 0.19\underline{/58^\circ}\ \Omega$$

The magnitude of the current in the HV is

$$\begin{aligned} |I| &= \frac{100{,}000}{440} \\ &= 227.27 \text{ A} \end{aligned}$$

Thus,

$$V_L = \frac{1}{2}(440\underline{/\beta_1^\circ} - IZ)$$

or

$$\begin{aligned} V_L\underline{/0^\circ} &= \frac{1}{2}[440\underline{/\beta_1^\circ} - 227.27\underline{/-36.9^\circ}\,(0.19\underline{/58^\circ})] \\ &= \frac{1}{2}[440\underline{/\beta_1^\circ} - 42.156\underline{/21.2^\circ}) \end{aligned}$$

Equating the imaginary components of the last equation, we obtain

$$\beta_1 = \arcsin \frac{15.25}{440} = 2^\circ$$

Equating the real components, we obtain

$$V_L = \frac{1}{2}(440 \cos 2^\circ - 39.3) = \underline{\underline{200.21 \text{ V}}}$$

b. Using Eq. (2.39), the regulation in percent is

$$\text{regulation } \% = \frac{220 - 200.21}{200.21}(100) = \underline{\underline{9.9\%}}$$

c. The short-circuit volt-amperes are given by

$$(\text{VA})_{sc} = V_{\text{rated}} I_{sc}$$

$$I_{sc} = \frac{V}{|Z|} = \frac{440}{0.19}$$

$$= 0.2372 \text{ A}$$

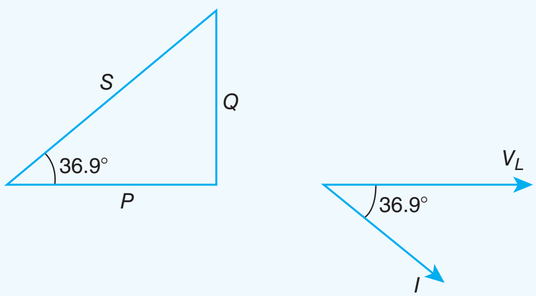

FIG. 2-23(c)

The short-circuit VA expressed in MVA is

$$(\text{MVA})_{sc} = 0.440(2.372)$$

$$= \underline{\underline{1.044 \text{ MVA}}}$$

Owing to the transformation properties of the transformer, the short-circuit MVA is the same whether it is referred to the high- or low-voltage winding. The power triangle and the phasor diagram of the load are shown in Fig. 2-23(c).

d. The required reactive power (Q_c) of the capacitor is

$$Q_c = P \tan \theta$$

from which

$$Q_c = 100(0.8)(\tan 36.9°)$$
$$= \underline{\underline{60 \text{ kVAR}}}$$

Exercise 2-2

The approximate equivalent circuit of a single-phase 5 kVA, 480–240 V transformer is shown in Fig. 2-24.

a. Draw the equivalent circuit as seen from the high-voltage side.
b. Draw the equivalent circuit as seen from the low-voltage side.
c. Find the per-unit value of the transformer's leakage impedance.
d. Determine the primary voltage when the transformer delivers rated current to a load at 240 volts and 0.80 power factor lagging.

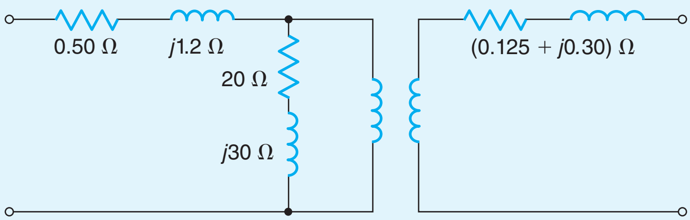

FIG. 2-24

Answer (a) $Z_{eH} = 1 + j2.4\ \Omega$; (b) $Z_{eL} = 0.25 + j0.6\ \Omega$; (c) $Z_e = (0.022 + j0.052)$ pu; (d) $503.52\angle 1.56°$ V.

Exercise 2-3

Show that the per-unit value of a transformer's equivalent leakage impedance is equal to the per-unit value of the voltage employed on the short-circuit test.

2.2 Three-Phase, Two-Winding Transformers

2.2.1 Introduction

Three-phase, two-winding transformers are used to interconnect two distribution systems of different voltages, such as a 25 kV transmission line and a 480 V plant. The majority of industrial transformers are of this type. They are called two-winding transformers because for each primary phase winding, there is only one secondary phase winding. The transformers actually have six coils—three for the incoming three-phase power and three for the output three-phase power.

Three-phase transformers can be either of the core type (Fig. 2-25) or of the shell type as shown in Fig. 2-26.

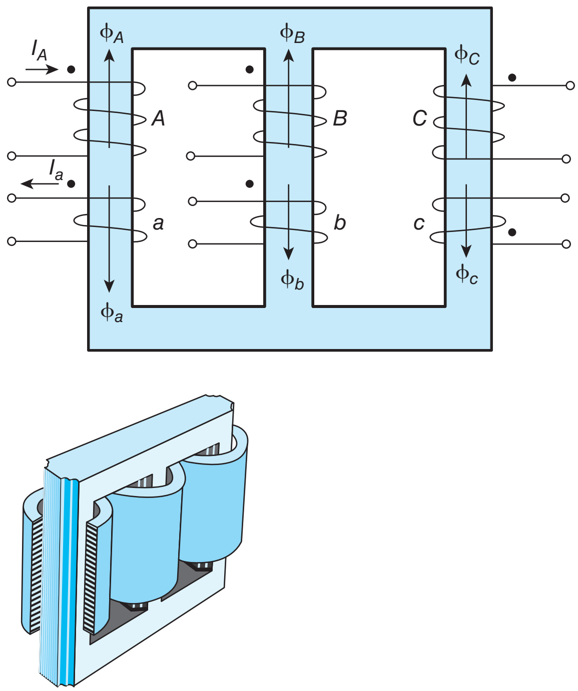

FIG. 2-25 Core-type, 3-ϕ, two-winding transformer.
Based on EPSTN, www.sayedsaad.com

Most transformers are of the core type. Their main advantage is that they prevent (ideally, if the phase core branches are identical and leakage is neglected) the presence of the third-harmonic flux, and hence, avoid inducing third-harmonic voltages. They are also less expensive than shell-type transformers of equivalent rating.

The prerequisites of understanding three-phase transformers are:

- Three-ϕ systems
- One-line diagrams

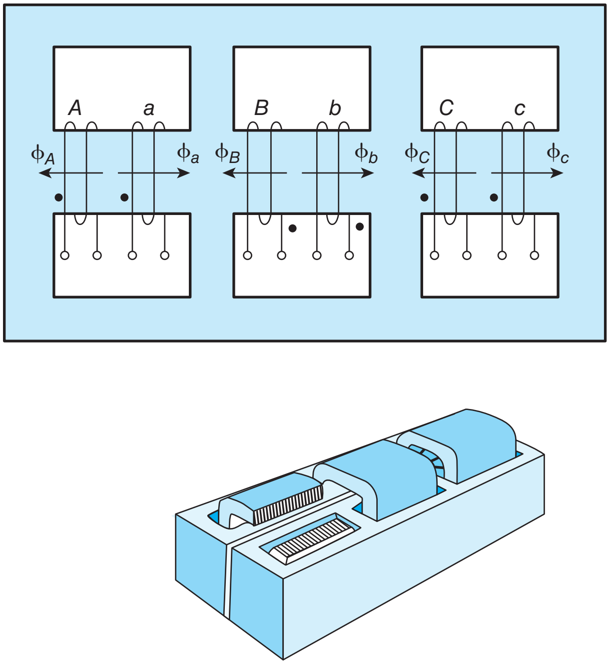

FIG. 2-26 Shell-type, 3-ϕ, two-winding transformer.
Based on EPSTN, www.sayedsaad.com

- Per-unit values
- Magnetic and electric circuit concepts
- Harmonics of the exciting current.

2.2.2 Review of Three-Phase Systems

Although three-phase systems are extensively discussed in Appendix A, they are briefly reviewed here for convenience.

Power Considerations

The active (P), reactive (Q), and complex power (S) drawn by a balanced three-phase load are given by

$$P = \sqrt{3} V_{L\text{-}L} I_L \cos \theta$$

$$Q = \sqrt{3} V_{L\text{-}L} I_L \sin \theta$$

$$S = \sqrt{3} V_{L\text{-}L} I_L^*$$

where $V_{L\text{-}L}$ is the rms value of the line-to-line voltage, I_L is the rms value of the line current, and I_L^* is its conjugate in phasor form. θ is the phase angle of the load impedance.

When analyzing a three-phase network, it is convenient to draw its power triangle. The real power is taken as reference, and the reactive power is 90 degrees out of phase with it. When the current lags the voltage, the power triangle is drawn leading. In other words, a lagging-power-factor load (which is characteristic of most industrial loads) draws positive VAR, and a leading-power-factor load (capacitive) draws negative VAR (references occasionally differ from this convention).

Current and Voltage Considerations

Depending on the type (Δ, Y) of three-phase connection, the phase and line parameters of a three-phase load are, as shown below, interrelated.

Delta-Connected Load

In a delta-connected load (Fig. 2.27), the line and phase voltages are equal, while the line current lags† the phase current by 30° and is larger by a factor of $\sqrt{3}$. Thus,

$$V_{L\text{-}L} = V_{p\text{-}p} \tag{2.47}$$

$$I_L = \sqrt{3} I_p \angle -30^\circ \tag{2.48}$$

where the subscript p stands for the phase parameters.

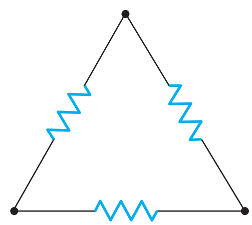

FIG. 2-27 Delta-connected load.

Star-Connected Load

In a star-connected load (Fig. 2-28), the line and phase currents are the same, while the line voltage leads† the phase voltage by 30° and is larger by a factor of $\sqrt{3}$. Thus,

$$I_L = I_p \tag{2.49}$$

$$V_{L\text{-}L} = \sqrt{3} V_{L\text{-}N} \angle 30^\circ \tag{2.50}$$

†The phase shift between the line and phase parameters depends on the order of rotation of the supply voltages (see Appendix A, Fig. A-2).

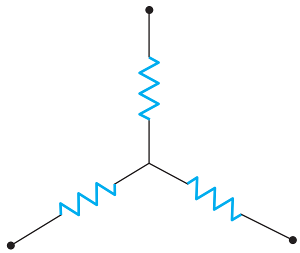

FIG. 2-28 Star-connected load.

Equations (2.48) and (2.50) are derived from a voltage phase sequence ABC.

The analysis of three-phase, two-winding transformers is simplified when their per-phase equivalent electric or magnetic circuits are used. The equivalent circuit of one phase is drawn, and the computations are identical to those used for single-phase transformers. Then the standard three-phase system relationships are employed to find the overall transformer parameters, such as currents, voltages, or power.

Phase Shift between Primary and Secondary Voltages

The phase shift, or the angular displacement, between the primary and the corresponding secondary voltage depends on the type of transformer connection. In a delta–delta or a star–star transformer, the angular displacement is zero; in a delta–star or a star–delta transformer, it is 30 degrees. According to the ASA,* the voltages on the HV side lead the corresponding voltages on the LV side by 30 degrees, regardless of which side is star or delta.

Magnetic Circuit Analysis

The magnetic circuit approach to the solution of three-phase transformer problems has the advantage of giving a physical insight into what is happening within the transformer. It can be of great assistance in computations involving the unbalanced operation of transformers.

A per-phase magnetic equivalent circuit is shown in Fig. 2-29(c). Neglecting the effects of the exciting current and applying KVL, we obtain

$$N_A I_A = N_a I_a \tag{2.51}$$

Also, from KCL, we get

$$\phi_A = \phi_a \tag{2.52}$$

As before, the upper-case subscripts represent the parameter of the primary side, while the lower-case subscripts represent the parameter of the secondary side.

*American Standard Association, C57.

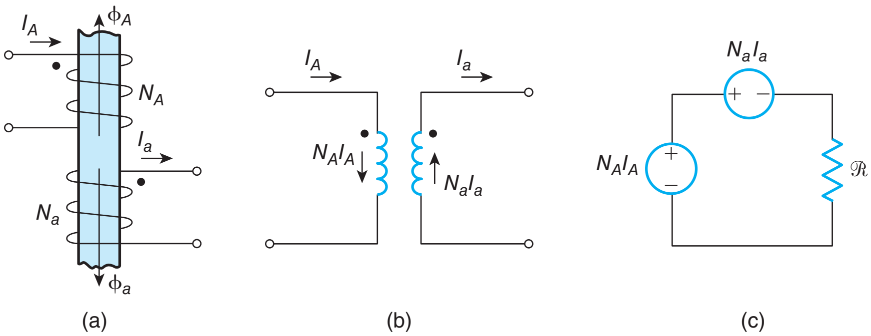

FIG. 2-29 Schematic for one phase of a two-winding, three-phase transformer. **(a)** Polarity of terminals and direction of flux. **(b)** Relative direction of mmf's. **(c)** Magnetic equivalent circuit per phase.

In analyzing problems, it is helpful if the coupled coils, with their actual magnetic volts, are drawn as shown in Fig. 2-29(b).

Electric Circuit Analysis

When electric circuit analysis is used to solve three-phase transformer problems, the following procedure is recommended:

1. Draw the per-phase electrical equivalent circuit.
2. Determine the turns ratio. This is given by the ratio of the phase to neutral voltages of the primary and secondary windings. This ratio is established by taking into consideration the delta- or star-type winding connection.
3. Draw the equivalent star–star connection of the given transformers for delta–star (Δ-Y) or delta–delta (Δ-Δ) type of winding connection. This is easily accomplished by using the standard delta–star equivalent impedances. That is,

$$Z_y = \frac{Z_\Delta}{3} \tag{2.53}$$

   where Z_Δ and Z_y represent, respectively, the equivalent impedances connected in delta and star. The above relationship is applicable only when the per-phase winding impedances are identical. When the winding impedances are unequal, changing a delta-connected load to its equivalent star is accomplished by using Eq. (A.6), found in Appendix A.
4. Select the reference voltage from the given data of the problem. Recall that when the transformer is connected delta–star or star–delta, the HV line-to-line voltages lead the corresponding LV line-to-line voltages by 30 degrees.

2.2.3 One-Line Diagram

Three-phase transformer coils can be connected in either star (Y) or delta (Δ) configurations. Figure 2-30(a) shows the industry's standard terminal identification for two-winding, three-phase transformers. The letters H and X represent high and low voltage, respectively, and the subscripts 1, 2, and 3 correspond to phases 1, 2, and 3, or phase rotation ABC.* The subscript 0 is used to designate the grounded terminal of a star-connected transformer. The neutral conductor of the system is also connected to this point when a four-wire distribution system is required. A star–star connected three-phase, two-winding transformer has, on the HV and LV windings, four terminals to which as many as five conductors can be connected.

The three conductors correspond to the 3-ϕ power cables, the fourth conductor constitutes the neutral of the system, and the fifth conductor is the ground of the transformer tank. The neutral is required when single-phase loads are to be supplied with line-to-neutral voltage. The ground and the neutral conductors are shown dotted in order to differentiate them from the other three main cables. The neutral is an insulated cable and carries current under normal operating conditions, while the ground conductor is usually not insulated and carries no current under normal operating conditions. However, when one of the phases of a grounded equipment is accidentally grounded, the resulting high short-circuit current will flow through the ground cable.

In the one-line diagram (Fig. 2-30(b)), the type of winding connections (Δ or Y) is shown. The first symbol indicates the primary or incoming terminal connection, and the second the outgoing or secondary terminal coil connection. The kVA, voltage, and impedance ratings are also marked on the one-line diagram.

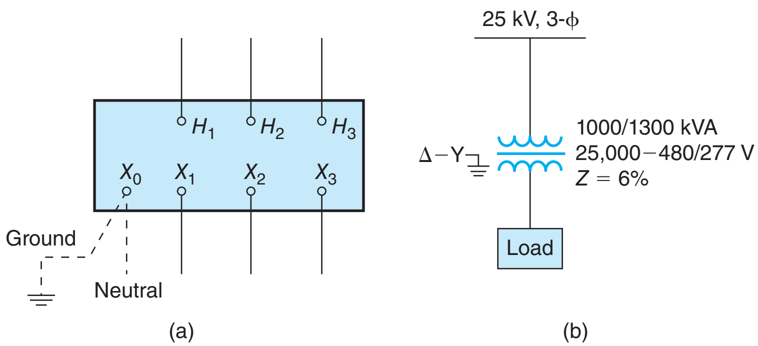

FIG. 2-30 Three-phase, two-winding transformers. **(a)** Terminal identification. **(b)** One-line diagram representation of a 1000/1300 kVA, 25,000–480/277 V.

*In practice, the phase rotation ABC is often designated as BRB. The latter expression identifies the black, red, and blue colors of the actual cables. When the ground wire is insulated, its color is green. The insulation of the neutral wire is normally white.

FIG. 2-31 Transformer with ventilation fans.
© *Markuso53|Dreamstime.com*

When the transformer is manufactured with special insulation, or equipped with fans, it can transform higher kVA without having its life expectancy reduced. Transformers equipped with one set of ventilating fans cost about 5% more but can deliver 33% more current at nominal voltages. Fig. 2-31 shows a substation transformer equipped with ventilating fans.

The per-unit impedance shown in the one-line diagram is calculated using the base kVA power of the transformer.

2.2.4 Types of Three-Phase Transformers

The following type of three-phase, two-winding transformers will be analyzed:

Δ-Y

Δ-Δ

Y-Δ

Y-Y

2-ϕ to 3-ϕ

1-ϕ to 3-ϕ

Delta–Star Transformers

Typical Δ-Y transformers are shown in Fig. 2-32 and Fig. 2-33. The Δ-Y type of three-ϕ transformers are very popular because they:

- Provide three-phase (*L-L-L*), one-phase (*L-L*), and *L-N* loads. The latter are necessary for computers, general usage receptacles, and the like.

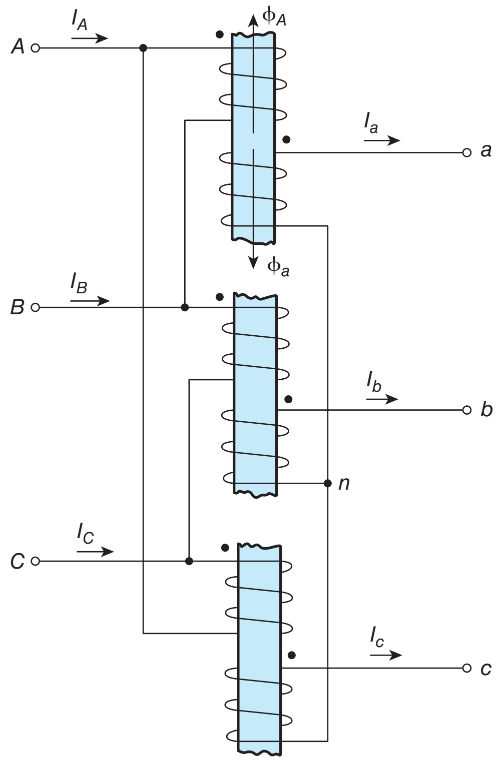

FIG. 2-32 Three single-phase transformers connected Δ-Y to form a two-winding, three-phase transformer bank.

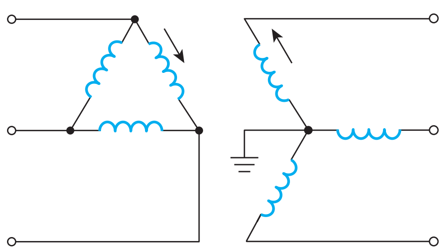

FIG. 2-33 Delta–star transformer.

- Provide means of grounding the neutral terminal which stabilize the reference voltage and can be used—through an impedance—to limit the L-G short-circuit current and thus contribute to mitigation of fires, danger to personnel, and cost of upstream current protective devices.
- Prevent the load's harmonic current components to circulate to the transformer's supply power lines.
- Are relatively more economical and have faster delivery.

Relatively speaking, the Δ-Y transformer maintains sinusoidal line-to-line waveforms in the primary and secondary windings.

The Δ-connected winding provides a short circuit to the flow of third harmonics. The zero-sequence* current components produced by the unbalanced loads, and the third harmonics of the nonsinusoidal magnetizing current are in phase for each winding and can circulate within the Δ-connected circuit.

The star-connected winding is subjected to $1/\sqrt{3}$ of the line-to-line voltage, and thus reduced insulation is required for the transformer windings.

One *disadvantage* of the Δ-Y transformer is that it cannot be paralleled with a Y-Y transformer. The transformer's line-to-line voltages are in phase in the high- and low-voltage sides, while the corresponding Δ-Y line-to-line voltages between primary and secondary are 30° apart. This phase difference is undesirable for parallel operation. It results in large circulating currents, which reduce the kVA output capability of the paralleled transformers.

Example 2-5

For the three-phase transformer shown in the one-line diagram of Fig. 2-34(a), determine:

a. The ohmic value of its reactance referred to the low- and high-voltage windings.

b. The magnitude of the short-circuit current in the transformer windings when a three-phase short takes place across the transformer's secondary terminals.

SOLUTION

a. The base parameters for the per-unit calculations are obtained from the rating of the transformer:

$$S_b = 1000\ \text{kVA}$$

$$V_{bH} = 25\ \text{kV}, \qquad V_{bL} = 480\ \text{V}$$

$$Z_{bH} = \frac{(25.0)^2}{1.0} = 625\ \Omega/\text{phase}, \qquad Z_{bL} = \frac{(0.48)^2}{1.0} = 0.23\ \Omega/\text{phase}$$

Z_{bH} is the equivalent star-connected impedance in Ω/phase. The base currents are

$$I_{bH} = \frac{1000}{\sqrt{3} \times 25} = 23.09\ \text{A}, \qquad I_{bL} = \frac{1000}{\sqrt{3}(0.48)} = 1202.81\ \text{A}$$

Then the ohmic values of the reactances are

HV Winding

$$X_H = 0.05(625) = \underline{\underline{31.25\ \Omega/\text{phase, star equivalent}}}$$

*The zero-sequence currents are due to unbalanced supply voltages and to nonidentical load impedances. They are similar to third harmonic currents.

LV Winding

$$X_L = 0.05(0.23) = \underline{\underline{0.012\ \Omega\text{/phase, star equivalent}}}$$

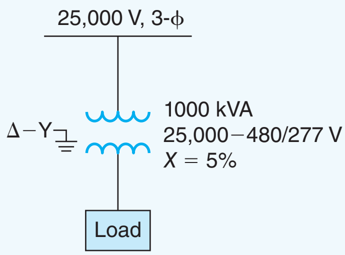

FIG. 2-34(a)

b. The short-circuit current can be found by using any of the following methods.

LV Winding

1. Applying Ohm's law in the circuit of Fig. 2-34(b), we find that the magnitude of the current in the LV winding is

$$I = \frac{V}{X} = \frac{480/\sqrt{3}}{0.012} = \underline{\underline{24{,}056.26\text{ A}}}$$

2. Using per-unit values:

$$I = \frac{1.0}{0.05} = 20\text{ pu}$$

$$= 20(1202.81) = \underline{\underline{24{,}056.26\text{ A}}}$$

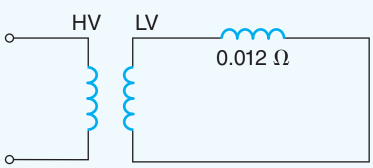

FIG. 2-34(b)

HV Side

1. Applying Ohm's law in the star equivalent circuit of Fig. 2-34(c), we obtain the magnitude of the short-circuit current in the HV windings:

$$I = \frac{V}{X} = \frac{25{,}000/\sqrt{3}}{31.25} = 461.88\text{ A line current}$$

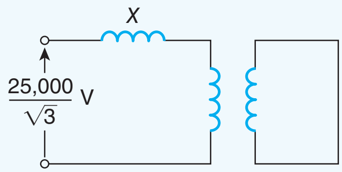

FIG. 2-34(c)

The magnitude of the phase current is

$$I_p = 461.88/\sqrt{3} = \underline{\underline{266.67\ \text{A}}}$$

2. Using Ohm's law, for the delta-connected equivalent circuit, we obtain

$$X_\Delta = 3X_Y = 3(31.25) = 93.75\ \Omega$$

$$I_p = \frac{25{,}000}{93.75} = \underline{\underline{266.67\ \text{A}}}$$

and

$$I_L = \sqrt{3}(266.67) = 461.88\ \text{A}$$

3. Refer to Fig. 2-34(d). By using the current transformation property of transformers, we obtain

$$I_p = 24{,}056.26\,\frac{480/\sqrt{3}}{25{,}000} = \underline{\underline{266.67\ \text{A}}}$$

and

$$I_L = \sqrt{3}I_p = 461.88\ \text{A}$$

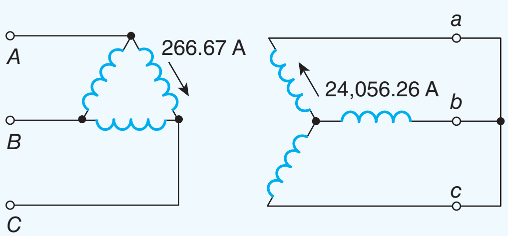

FIG. 2-34(d)

EXAMPLE 2-6

Three identical single-phase transformers, each of 500 kVA, 4160–480 V, 60 Hz, are connected Δ-Y to form a 3-ϕ transformer bank. Data for the short-circuit test on one of the single-phase transformers is as follows:

Low-voltage winding shorted:

$$V_H = 250\text{ V}, \qquad I_H = 120.19\text{ A}, \qquad P = 10\text{ kW},$$

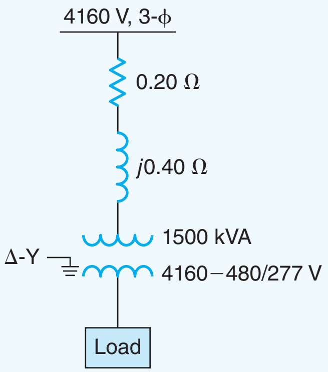

FIG. 2-35(a)

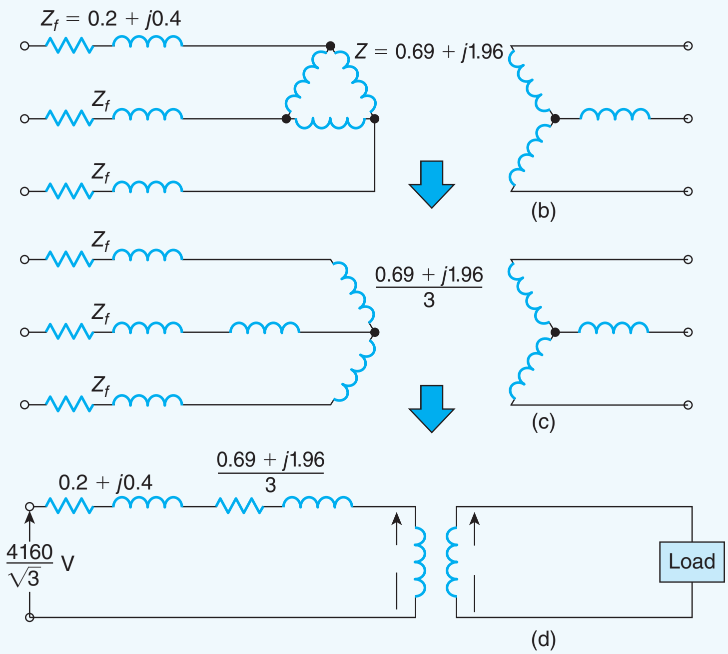

FIG. 2-35 (b), (c), and (d)

The transformer, as shown in Fig. 2-35(a), is connected to an upstream substation of 4160 volts through a feeder whose impedance is $(0.2 + j0.4)$ ohms per phase. When the transformer delivers rated current at 0.9 power factor lagging, determine:

a. The line-to-line voltage across the load.
b. The power supplied by the upstream substation.
c. The current through the transformer windings when a 3-ϕ short takes place at the secondary terminals of the transformer.

SOLUTION

a. From the short-circuit test data, we get

$$|Z_{eH}| = \frac{250}{120.19} = 2.08\ \Omega/\text{phase}$$
$$R_{eH} = \frac{10{,}000}{(120.19)^2} = 0.69\ \Omega/\text{phase}$$
$$X_{eH} = \sqrt{2.08^2 - 0.69^2} = 1.96\ \Omega/\text{phase}$$

Thus,

$$Z_{eH} = 0.69 + j1.96$$
$$= 2.08\underline{/70.6^\circ}\ \Omega/\text{phase}$$

This is the per-phase impedance referred to the HV winding, which is delta connected. In order to simplify the calculations of the primary, as shown in Fig. 2-35(c), it is changed to an equivalent star. Then the per-phase equivalent circuit, as seen from the HV winding, is shown in Fig. 2-35(d).

The magnitude of the line current in the primary of the Δ-Y transformer is

$$I = \frac{1500}{\sqrt{3}(4.160)} = 208.18\ \text{A}$$

Refer to Fig 2-35(d). The total impedance in the primary circuit is

$$Z = 0.2 + j0.4 + \frac{0.69 + j1.96}{3} = 1.14\underline{/67.8^\circ}\ \ \Omega/\text{phase}$$

Applying KVL to the per-phase equivalent circuit of Fig 2-35(d), we obtain

$$V_1\underline{/0^\circ} = \frac{4160}{\sqrt{3}}\underline{/\beta_1} - 208.18\underline{/-25.8^\circ}\,(1.14\underline{/67.8^\circ})$$
$$= \frac{4160}{\sqrt{3}}\underline{/\beta_1} - 237\underline{/-41.9^\circ}$$

Equating the imaginary parts, we obtain

$$\beta_1 = \arcsin \frac{\sqrt{3}}{4160} (237 \sin 41.9°) = 3.8°$$

Equating the real parts, we get

$$V_1 = \frac{4160}{\sqrt{3}} \cos 3.8° - 237 (\cos 41.9°) = 2220.21 \text{ V/phase}$$

The voltage across the secondary of the transformer is

$$V_2 = V_1 \left(\frac{480}{4160}\right) = 2220.21 \frac{480}{4160} = 256.18 \text{ V/phase}$$

$$= (256.18)\sqrt{3} = \underline{\underline{443.71 \text{ V}_{L\text{-}L}}}$$

b. The power supplied is

$$P = \sqrt{3} V_{L\text{-}L} I_L \cos \theta$$

$$= \sqrt{3}(4160)(208.18) \cos (3.8° + 25.8°)$$

$$= 1303.96 \text{ kW}$$

c. The magnitude of the short-circuit current through the feeder is

$$I_L = \frac{V}{Z} = \frac{4160}{\sqrt{3}(1.14)} = 2109.60 \text{ A}$$

The current in the Δ-connected windings is

$$I_P = \frac{I_L}{\sqrt{3}} = \frac{2109.60}{\sqrt{3}} = \underline{\underline{1218 \text{ A}}}$$

The current in the Y-connected windings is

$$I_s = (1218) \frac{4160}{480/\sqrt{3}} = \underline{\underline{18{,}283.20 \text{ A}}}$$

Alternatively, using per-unit values,

$$Z_{bH} = \left(\frac{4160}{1.5}\right)^2 = 11.54 \ \Omega\text{/phase}$$

Thus,

$$Z_{e\,pu} = \frac{1.14}{11.54} = 0.099\text{ pu}$$

$$I_{sc} = \left(\frac{1}{0.099}\right) = 10.13\text{ pu}$$

In the LV side:

$$I_{sc} = 10.13\,\frac{1500}{\sqrt{3}(0.480)} = \underline{\underline{18{,}283.20\text{ A}}}$$

In the HV side:

$$I_{sc} = 10.13\,\frac{1500}{\sqrt{3}(4.16)} = \underline{\underline{2109.60\text{ A}}}$$

Exercise 2-4

The three-phase transformer shown in the one-line diagram of Fig. 2-36 is connected to a 4.16 kV source. The magnetizing current is negligible.

1. If the transformer delivers its base kVA at the rated voltage, determine:
   a. The primary and secondary line and phase currents.
   b. The ohmic values of the leakage reactance referred to the HV and LV windings.
   c. The primary and secondary line and phase currents when the secondary terminals of the transformer are shorted.
2. Repeat (a), (b), and (c) when the transformer operates with the fans and delivers 2000 kVA at rated voltage.

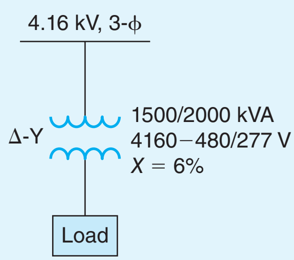

FIG. 2-36

Answer 1. (a) 1804.2 A, 208.18 A, 120.2 A
(b) 2.08 Ω, 0.0092 Ω/ph
(c) 30,070.3 A, 2003.2 A, 3469.6 A
2. (a) 2405.6 A, 277.6 A, 160.3 A
(b) and (c) no change

Exercise 2-5

The three-phase, three-winding transformer shown in Fig. 2-37 delivers 25 A to a single-phase load. The tertiary winding (T) is shorted, and the coils have an equal number of turns.

a. Mark the polarity of each coil and show the direction of its flux.
b. Determine the current of each winding. (*Hint*: The net flux through each leg must be equal to zero.)

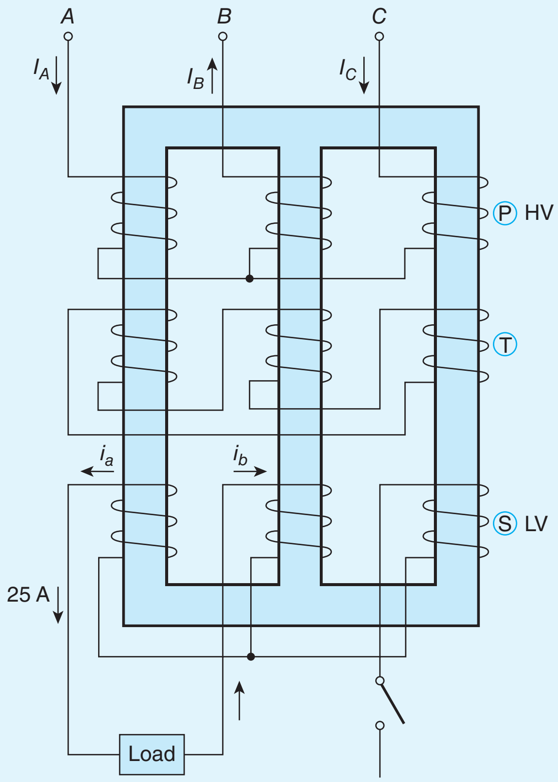

FIG. 2-37

Answer $I_T = 0,\ i_A = i_B = 25$ A, $i_C = 0$

Δ-Δ *Transformer*

A Δ-Δ winding connection is shown in Fig. 2-38. This type of transformer has the following advantages:

1. When a line-to-ground fault—the most common type of fault—occurs in the secondary distribution of the transformer, the resulting current is very small and therefore the power flow is not interrupted. This is of tremendous importance to industries whose processes—such as the melting of metals—cannot be interrupted.
2. This type of transformer connection maintains sinusoidal voltages at its output terminals. This is because the delta–delta winding connection provides a path for the flow of the third harmonic component of the magnetizing current and for the circulation of the zero-sequence component of the unbalanced load currents.
3. When three single-phase transformers are connected in Δ-Δ to form a 3-ϕ transformer bank, one of the single-phase transformers can be removed and the resulting open-delta or VEE transformer bank can deliver about 58% of its original rating to a three-phase secondary load.

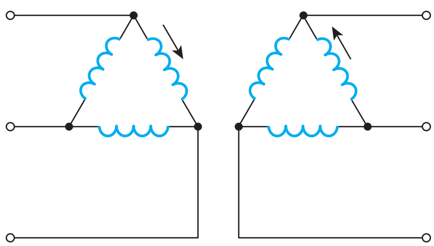

FIG. 2-38 Delta–delta transformer.

In such operations, the transformer is referred to as a V-type or open-delta type of transformer.

From the theory of three-phase circuits, the per-phase current (I_P) is related to the line current (I_L) by the $\sqrt{3}$. That is,

$$I_P = \frac{I_L}{\sqrt{3}}$$

When one of three-single-phase transformers fails, the secondary winding phase currents are equal to line currents that were originally designed to carry only 58% of the line current.

The additional advantage of the delta–delta transformer is that it will continue to operate while there is a line-to-ground short circuit.

The disadvantage of this type of winding configuration is that it cannot supply L–N single-phase loads.

Star–Delta Transformer

The Y-Δ transformer is of limited use because it does not provide a grounded neutral and thus does not support L-N, single-phase loads.

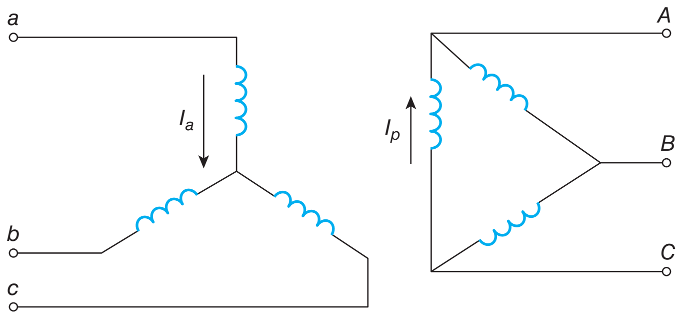

FIG. 2-39 Y-Δ Transformer.

Star–Star Transformer

The Y-Y transformer, when the star-point is grounded, can provide L–L–L, L–L, and L-N loads. However, it permits the flow of harmonic current through the upstream and downstream network.

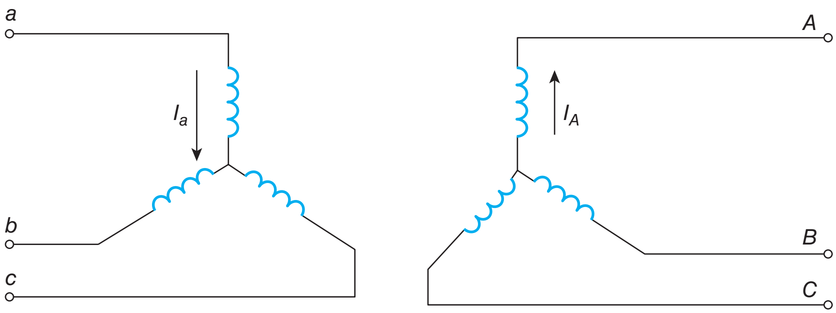

FIG. 2-40 Y-Y Transformer.

An ungrounded Y-Y transformer is shown in Fig. 2-41. When the transformer's load is slightly unbalanced, the neutral points (n) may not be at a fixed potential with respect to ground. This is referred to as the "floating" neutral. The potentially unstable neutral is due mainly to unbalanced load currents.

Considering Fig. 2-41, from KVL, in the primary side, we have

$$V_{Cn} + V_{nA} + V_{AC} = 0$$

or

$$V_{AC} = V_{nC} + V_{An}$$

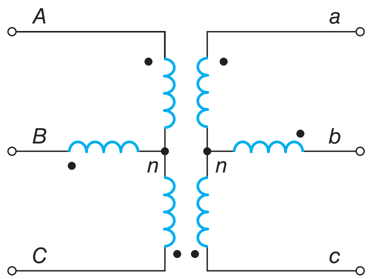

FIG. 2-41 Ungrounded star–star (Y-Y) connected transformer.

The potential V_{AC} is constant, owing to the primary voltage source. The other voltages (V_{nC} and V_{An}) will vary in a way that satisfies the previous two equations.

To stabilize the potential of the neutral point, a delta-connected winding—the so-called tertiary—is wound on the core of the transformer. In this case, the transformer becomes a three-phase, three-winding transformer. Such transformers have high MVA ratings and are normally used by the utilities.

EXAMPLE 2-7

Show that, when a single-phase load is connected from one line to neutral in the secondary of an ungrounded Y-Y, 3-ϕ transformer, the voltage across the loaded phase is reduced while the other voltages from line-to-neutral will be increased.

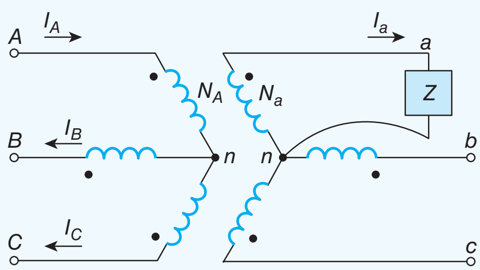

FIG. 2-42

SOLUTION

Assume a load is connected from line "a" to neutral, as shown in Fig. 2-42. The load impedance will draw a current I_a because of the voltage induced in the secondary winding:

$$I_a = \frac{V_a}{Z}$$

Since there is a current on the secondary means, there must also be a current I_A in the primary, because of the current-transformation property of transformers, namely:

$$N_A I_A = N_a I_a$$

Applying KCL, in the primary winding, we have

$$I_A = I_B + I_C$$

I_A exists, and, since $I_b = I_c = 0$, then I_B and I_C must represent increases in the exciting currents in phases B and C, respectively. Thus, their corresponding voltages V_{Bn} and V_{Cn} must also be *increased*. This leads to the reduction of V_{AB} ($V_{AB} = V_{nB} + V_{An}$) and V_{ab}.

Exercise 2-6

Compare and contrast the Δ-Δ and Δ-Y type of three-phase, two-winding transformers. Consider insulation levels, exciting currents, and output voltage waveforms.

Special Type of Transformers

In this section we will describe the transformers that change a 2-ϕ and 1-ϕ voltage supply to a 3-ϕ balanced system. These special transformers may be analyzed by drawing their corresponding magnetic equivalent circuits in terms of their mmf's.

Scott-Connection

A scott-transformer connection, as shown in Fig. 2-43, is used to interconnect a 2-ϕ system to a 3-ϕ distribution network. The 2-ϕ voltages are equal in magnitude and at 90 electrical degrees to each other.

Such transformers have two cores, and in the 3-ϕ side, one of the windings has 0.866 N number of turns and is connected to the center tap of the other winding, which has N number of winding turns. This winding arrangement insures a 120-degree phase shift among the voltages of the 3-winding.

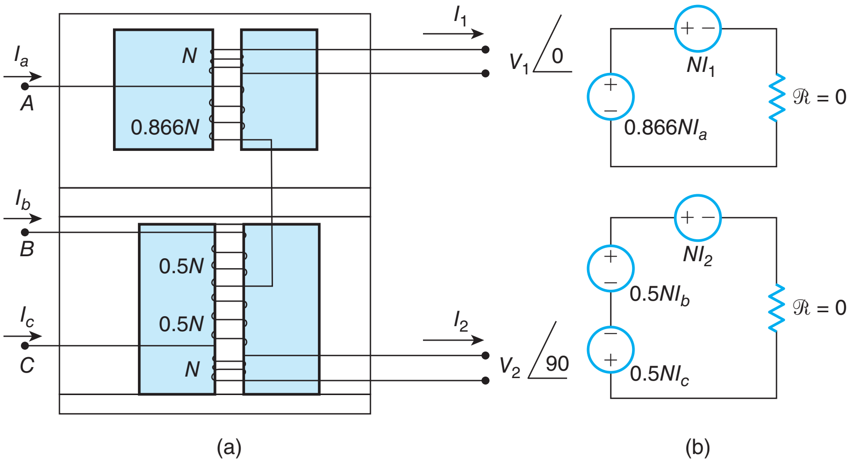

FIG. 2-43 Scott-type of transformer. (a) Winding configuration. (b) Equivalent magnetic circuits.

Refer to Fig. 2-43. Assuming an equal number of winding turns (N), we have:

Winding One

Neglecting the core losses and applying KVL in the magnetic circuit of winding 1, we obtain

$$NI_1 = 0.866\, NI_a$$

or

$$NI_1 = \underline{\underline{0.866\, NI_a \angle 0}}$$

Similarly, for winding 2, we obtain

$$NI_2 + 0.5\, NI_b - 0.5\, NI_c = 0$$

or

$$NI_2 = 0.5N(I_c - I_b)$$

The magnitudes of the line currents I_a, I_b, and I_c are equal:

$$NI_2 = 0.5NI_a\,(1\angle 120 - 1\angle -120)$$
$$= \underline{\underline{0.866NI_a \angle 90}}$$

Thus the mmf's on the two windings are equal in magnitude and at 90 electrical degrees to each other.

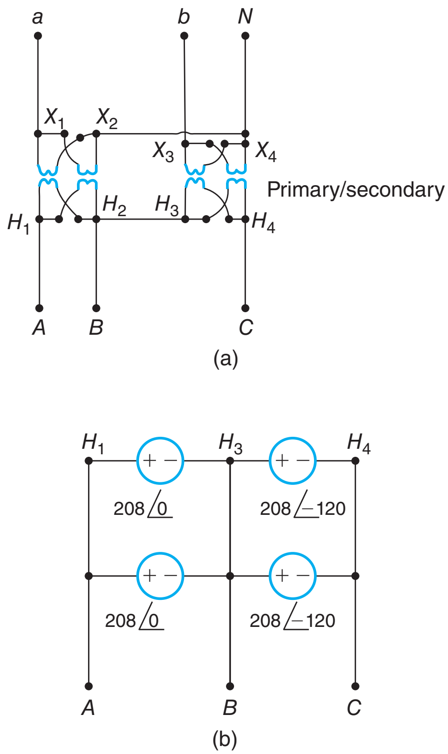

FIG. 2-44 One-phase to three-phase transformation. (a) Winding configuration. (b) Equivalent electrical circuit (ideal).

(The mmf's are proportional to magnetic flux whose derivative is proportional to a corresponding voltage).

1-ϕ to 3-ϕ Transformation

Figure 2-44 shows the winding configuration of a special transformer that is used to change a single-phase line-to-line voltage to a three-phase balanced voltage supply.

These types of transformers are for economical reasons used within a plant to supply a remote 3-ϕ panel. The savings result by using two wires instead of three wires.

For a 208 V line-to-line supply voltage, we have

$$V_{ab} = 208\underline{/30}\ \text{V}, \qquad V_{an} = 120\underline{/0} \quad \text{and} \quad V_{bn} = 120\underline{/-120}\ \text{V}$$

The transformer's winding turns ratio is such as to develop 208 V across the secondary windings. That is,

$$V_{AB} = 208\underline{/0}, \qquad V_{BC} = 208\underline{/-120}\ \text{V}$$

The electrical equivalent circuit is shown in Fig. 2-42(b). For simplicity,

$$K = 208 \text{ V}$$

The voltage V_{CA} can be obtained graphically or by applying KVL in the loop:

$$\text{C} - \text{A} - \text{H1} - \text{H2} - \text{H3} - \text{H4} - \text{C}$$

Thus,

$$V_{CA} + K\angle 0 + K\angle -120 = 0$$

from which

$$\begin{aligned} V_{CA} &= K\angle 120 \\ &= 208\angle 120 \text{ V} \end{aligned}$$

Due to leakage impedances and a slightly unequal turns ratio, there would be circulating currents. As a result, the efficiency of such transformers is lower than that of their equivalent standard units.

2.2.5 Harmonics of the Exciting Current

This section discusses the harmonics of the exciting current and the advantages and disadvantages of the various transformer winding connections.

Harmonics of the Exciting Current

Consider the exciting current of one phase only. Neglecting its fourth and higher-order harmonics, we have

$$\begin{aligned} i_{\text{exc}} &= i_1 + i_2 + i_3 \\ &= I_{m1} \cos \omega t + I_{m2} \cos 2\omega t + I_{m3} \cos 3\omega t \end{aligned} \qquad \textbf{(2.54)}$$

where I_{m1}, I_{m2}, and I_{m3} are the maximum values of the fundamental, second, and third harmonics.

Second Harmonics

Designating I_{m2} as the maximum value of the second harmonic in one phase, we see that the vectorial sum for all three phases is

$$i_2 = I_{m2} \cos 2\omega t + I_{m2} \cos 2(\omega t + 120°) + I_{m2} \cos 2(\omega t + 240°) \qquad \textbf{(2.55)}$$

From the above:

$$i_2 = 0$$

Thus, the sum of the second, or even, harmonics of a three-phase system is equal to zero, regardless of the transformer winding connections.

Third Harmonic

The third harmonic components of the exciting current in the lines of a three-phase system do not add up to zero, as in the case of the fundamental and second harmonics. Mathematically,

$$i_3 = I_{m3} \cos 3\omega t + I_{m3} \cos 3(\omega t + 120°) + I_{m3} \cos 3(\omega t + 240°) \tag{2.56}$$

From the above,

$$i_3 = 3I_{m3} \cos 3\omega t \tag{2.57}$$

When third harmonics are permitted to flow, the exciting current is nonsinusoidal in waveform. This waveform—as can be verified by the *B-H* characteristic of high-permeability magnetic materials—is accompanied by sinusoidal flux. Sinusoidal flux produces sinusoidal secondary voltages, and nonsinusoidal flux produces nonsinusoidal secondary voltages. These nonsinusoidal voltages are undesirable because they may cause electrical apparatuses—motors, computers, and the like—to malfunction. Thus, it is desirable that the waveform of the exciting current be nonsinusoidal.

The nonsinusoidal waveform of the exciting current is ensured when the transformer connections permit the flow of third harmonics. Star-connected windings with a grounded neutral provide a physical link so that the harmonics can flow from the system's lines down to ground, then eventually return to the generator's grounded neutral. Therefore, star-grounded windings permit the flow of nonsinusoidal excitation currents, and so their secondary phase voltages are sinusoidal. Delta-connected windings also produce sinusoidal fluxes and voltages because they permit the third harmonics to circulate within their windings.

In contrast, ungrounded star-connected windings do not permit the flow of third harmonics of exciting currents, and thus the induced voltages are nonsinusoidal.

2.3 Autotransformers

This section covers the principles of operation, equivalent circuits and losses. Figure 2-45(a) shows the schematic representation of a 10 kVA, 200–100 V, single-phase transformer. Its autotransformer-type connection can be obtained as shown in Figs. 2-45(b) and (c), by a special interconnection of its primary and secondary windings.

In autotransformers, one single winding is used as the primary winding as well as secondary winding, as against two distinctly separate windings in a conventional power transformer. Autotransformers are smaller in size and more economical than two-winding transformers of the same rating. They are normally used for voltage-transformation ratios close to 1:1 and in variacs, which provide a variable secondary voltage.

Understanding and analyzing autotransformers is simplified by noting the following:

1. The ampere-turns of each coil are the same whether the transformer is connected as an autotransformer or as a two-winding transformer. That is,

$$(NI) = \text{constant} \tag{2.60}$$

   In other words, the magnitude of the current through each coil remains the same, regardless of whether the transformer operates as an autotransformer or as a conventional two-winding transformer.

   Referring to Fig. 2-45, the ampere-turns of each coil are the same in each of the three situations shown, but the complex power transferred from the supply to the load differs from one case to the next.

2. Since the current through each autotransformer winding is the same as in the conventional two-winding transformer, the winding loss remains the same. However, the efficiency of the autotransformer is increased if the output power is increased.

3. Neglecting losses, the complex power (kVA) at the input is equal to the complex power delivered to the output:

$$(\text{kVA})_{\text{input}} = (\text{kVA})_{\text{output}} \tag{2.61}$$

   The kVA transformation capability of the autotransformer is the same as that of the two-winding transformer. An autotransformer, however, delivers higher kVA than the conventional transformer because of the direct electrical connection between the primary and secondary windings. In other words, part of the output kVA is conducted from the primary to the secondary winding. The conducted kVA is referred to as untransformed kVA.

4. The two-winding conventional transformer has its primary and secondary circuits electrically isolated, while in the autotransformer electrical disturbances in the primary can be easily passed to the secondary through their direct electrical connection.

A two-winding transformer, as shown in Fig. 2-45, can be connected in two ways to supply a load as an autotransformer. The two different connections are identified as A and B. For reasons of comparison, the highlights of type A and B connections are summarized in Table 2-1. In both cases, the output voltage is smaller than the input voltage. However, when one of the voltage windings is reversed, the output voltage will be larger than the input voltage.

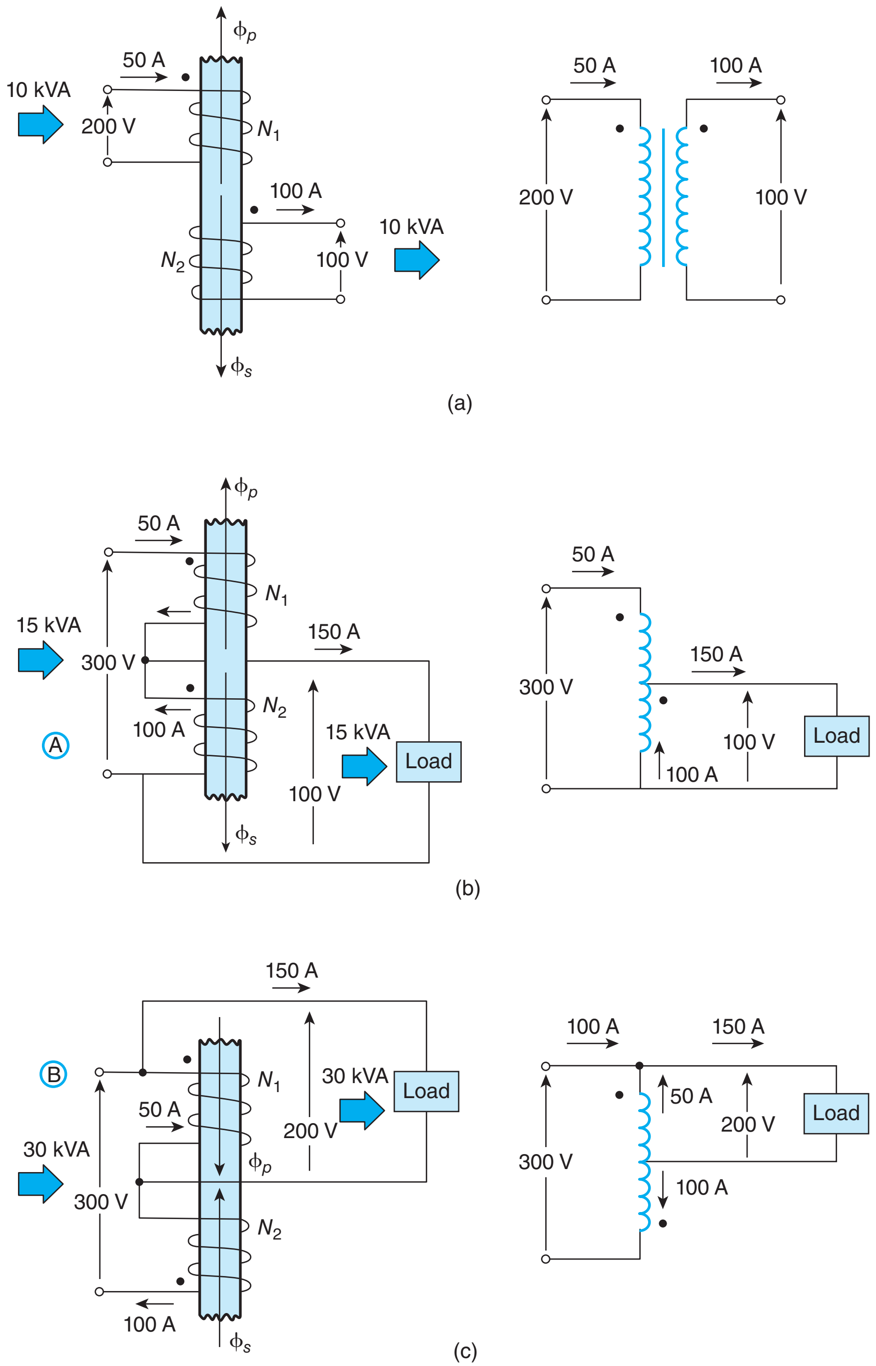

FIG. 2-45 Transformer representations and their schematics. (a) A two-winding transformer. (b) Type A autotransformer connection. (c) Type B autotransformer connection.

TABLE 2-1 Parameters of two-winding transformers and autotransformers

Parameter	*Conventional Two-Winding Transformer*	*Autotransformer Connection Type A: Load across the Same Winding as in the Two-Winding Transformer*	*Autotransformer Connection Type B: Load Connected across the Primary Winding of the Two-Winding Transformer*
Turns ratio	$a = \frac{N_1}{N_2}$	$a + 1$	$\frac{a+1}{a}$
Winding voltage	V	V	V
Winding current	I	I	I
Efficiency	High	Higher	Much higher
Input short-circuit current	I_{sc}	$\frac{(1+a)}{a} I_{sc}$	$a(a+1)I_{sc}$
kVA transformed	S	S	S
kVA untransformed (conducted)	0	$\frac{S}{a}$	aS
Total kVA Output	S	$S\frac{(a+1)}{a}$	$S(a+1)$

Example 2-8

A 480–120 V, 20 kVA, two-winding transformer with a leakage impedance of $(0.02 + j0.05)$ per unit has an efficiency of 0.96 when it delivers rated current at 0.90 power factor lagging. This transformer is to be connected as an autotransformer to a 600 V source to supply a load at either 480 or 120 V. For each *type* of autotransformer connection determine:

a. The kVA rating of the autotransformer. What percent of this kVA passes through the autotransformer untransformed?

b. The efficiency at full-load and 0.90 power factor lagging.

c. The input current when the load is shorted. Compare the results with those of the two-winding operation.

SOLUTION

This problem compares the essential parameters of a two-winding transformer with those of its autotransformer connections.

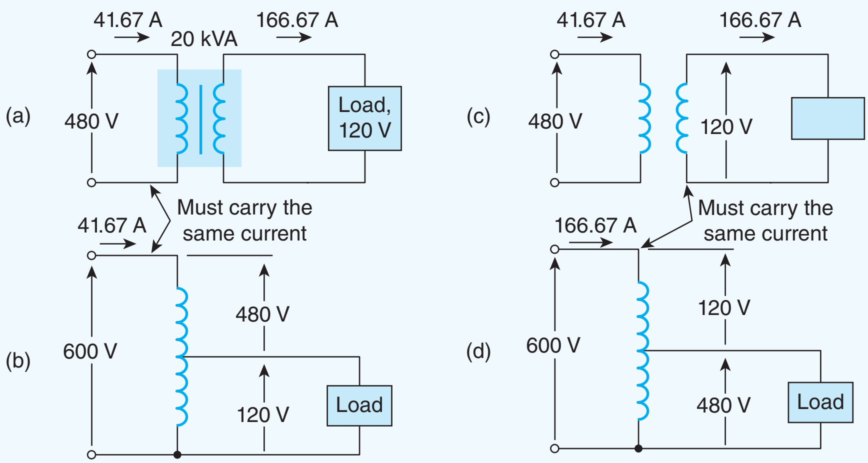

FIG. 2-46 (a) and (b): Two-winding and autotransformer connection (type A). (c) and (d): Two-winding and autotransformer connection (type B).

Type (A) Autotransformer Connection (Load across the 120 V Winding)

a. Referring to Fig. 2-46(b), the magnitude of the current through the input will be the current rating of the 480 V winding. That is,

$$I = \frac{20{,}000}{480} = 41.67 \text{ A}$$

Therefore, the input apparent power to the autotransformer is

$$S = 41.67(600)$$
$$= \underline{\underline{25 \text{ kVA}}}$$

Of these 25 kVA, 5 kVA pass unaltered, or untransformed, from input to output; the remaining 20 kVA are transformed, as in the case of the two-winding transformer connection. Thus,

$$\text{percentage of untransformed kVA} = \frac{5}{25}(100)$$
$$= \underline{\underline{20\%}}$$

b. The losses of the two-winding transformer are

$$\text{losses} = \text{power input} - \text{power output} = \frac{P_{\text{out}}}{\eta} - P_{\text{out}}$$

$$= 20{,}000(0.9)\left(\frac{1}{0.96} - 1\right) = 750 \text{ W}$$

For the autotransformer connection, the losses remain constant. Thus, the efficiency is

$$\eta = \frac{25(0.90)}{25(0.90) + 0.750} = 0.97$$

c. For the two-winding transformer, the magnitude of the short-circuit current is

$$I_{sc} = \left|\frac{1}{0.02 + j0.05}\right|$$
$$= 18.57 \text{ pu}$$

For the HV winding, the current in amperes is

$$I_{sc} = 18.57(41.66)$$
$$= \underline{\underline{773.73 \text{ A}}}$$

The leakage impedance, in ohms, is

HV side

$$Z_{bH} = \frac{(480)^2}{20{,}000} = 11.52\ \Omega$$
$$Z_H = (0.02 + j0.05)(11.52) = 0.62\angle 68.2^\circ\ \Omega$$

LV side

$$Z_{bL} = \frac{(120)^2}{20{,}000} = 0.72\ \Omega$$
$$Z_L = (0.02 + j0.05)(0.72) = 0.04\angle 68.2^\circ\ \Omega$$

In the case of the autotransformer, when the 120 V winding is shorted, the source of 600 V is applied across the 480 V winding. Thus, the magnitude of the short-circuit current is

$$I_{sc} = \frac{600}{0.62} = \underline{\underline{967.16 \text{ A}}}$$

or, by using the appropriate relationship from Table 2-1.

$$I_{sc} = 773.73\frac{(1 + a)}{a} = 773.73\left(\frac{5}{4}\right)$$
$$= \underline{\underline{967.16 \text{ A}}}$$

Type (B) Autotransformer Connection (Load across the 480 V Winding)

a. Referring to Fig. 2-46(d), the current through the 600 V source will be the rated current of the 120 V winding. That is,

$$I = \frac{20 \times 10^3}{120} = 166.67 \text{ A}$$

This current must pass through the input terminals. Therefore, the apparent input power for this type of autotransformer connection is

$$S = 166.67(600)$$
$$= \underline{\underline{100}}\text{ kVA}$$

Of these 100 kVA, 80 kVA pass unaltered from the input to the output, while the remaining 20 kVA are transformed as in the case of the two-winding connection.

Expressing the untransformed kVA in percentage of the total output kVA, we have

$$(kVA)_{\text{untr.}} = \frac{80}{100}(100) = \underline{\underline{80\%}}$$

b. Since the losses are constant at 0.75 kW, the efficiency is

$$\eta = \frac{100(0.9)}{100(0.9) + 0.75}$$
$$= \underline{\underline{0.99}}$$

c. By shorting the load, the 600 V are applied across the 120 V winding. Thus, the magnitude of the short-circuit current is

$$I_{sc} = \frac{600}{0.04} = \underline{\underline{15{,}474.6\text{ A}}}$$

For comparison purposes, the results are summarized in Table 2-2. Check the results by using the applicable relationships given in Table 2-1.

TABLE 2-2 Summary of results of Example 2-8

		Autotransformer Connection	
Parameter	*Two-Winding Transformer Connection*	*Type A Load across 120 V*	*Type B Load across 480 V*
Output kVA	20	25	100
Efficiency	0.96	0.97	0.99
Short-circuit current through the source in A	773.7	967.16	15,474.6

Exercise 2-7

A single-phase, 1000 VA, 60 Hz autotransformer has four coils, as shown in Fig. 2-47. The voltage rating of the HV windings is 240 V/coil, and that of the LV windings is 120/coil. Show the field connections, and determine the maximum VA that can be delivered to the load when:

a. The transformer supplies 120 V from a 240 V source.
b. The transformer supplies 240 V from a 480 V source.
c. Two of these transformers are connected in an open-delta configuration to supply a 3-ϕ, 480 V load from a 3-ϕ, 600 V source.

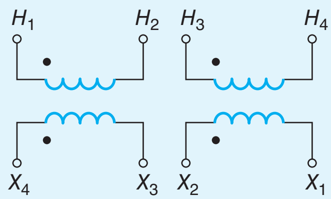

FIG. 2-47

Answer (a) 1000 VA; (b) 1000 VA; (c) 8660 VA

Disadvantages of Autotransformers

The disadvantages of autotransformers are as follows:

- They are not economical for voltage rations larger than 2.
- They require primary protective devices of higher capacity.

1. Because of the electrical conductivity of the primary and secondary windings, the lower voltage circuit is liable to be impressed upon by higher voltage. To avoid breakdown in the lower voltage circuit, it becomes necessary to design the low-voltage circuit to withstand higher voltage.
2. The autotransformer has a common terminal between the primary and the secondary windings, and when the secondary is shorted, the voltage applied to the primary is much higher than its rated voltage. This results in higher short-circuit currents and thus requires more expensive protective devices.
3. The connections on primary and secondary sides must necessarily be the same, except when using interconnected starring connections. This introduces complications due to changing primary and secondary phase angles, particularly in the case-by-case of the delta–delta connection.
4. Because a common neutral in a star–star connected autotransformer, it is not possible to ground the neutral of one side only. Both of its sides must have its neutrals either grounded or isolated.

5. It is more difficult to preserve the electromagnetic balance of the winding when voltage adjustment tappings are provided. It should be known that the provision of adjusting tapping on an autotransformer increases the frame size of the transformer considerably. (Source: http://www.electrical4u.com/electrical-transformer/auto-transformer.php)

2.4 Parallel Operation of Transformers

Transformers are sometimes paralleled in order to meet the increased kVA demands of a particular load. Transformers that are to be paralleled satisfactorily should satisfy the following conditions:

a. Equal impedances.
b. Equal turns ratio.
c. Equal phase shift between the primary and secondary open-circuited voltages.
d. The same phase rotation.

When these conditions are not satisfied, the transformers—as is demonstrated below—will be damaged.

The one-line representation of parallel transformers, as well as their corresponding equivalent circuits, are shown in Fig. 2-48. For the analysis of paralleled

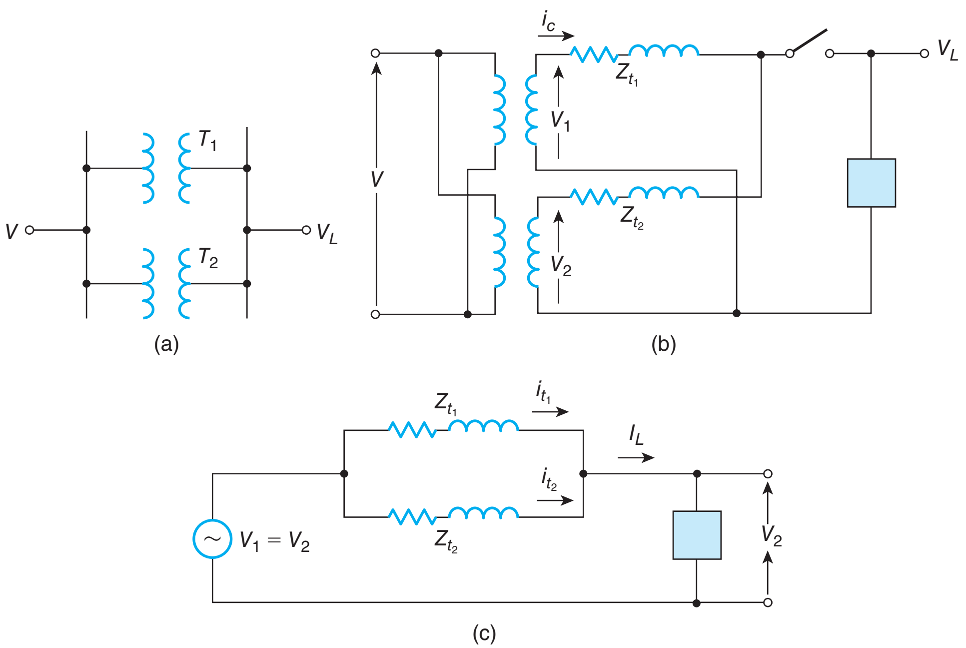

FIG. 2-48 Two single-phase transformers in parallel. (a) One-line diagram. (b) Equivalent circuit for determining the circulating current. (c) Equivalent circuit for determining the load on each transformer. (The turns ratio for all units are assumed to be equal.)

transformers, the per-phase equivalent circuits of Fig. 2-48(b) and Fig. 2-48(c) can be used.

In Fig. 2-48(b), the load is disconnected from the common secondary transformer terminals, and thus this circuit can be used to find the transformer's common circulating current. When the transformers have an equal number of winding turns, then Fig. 2-48(c) may be used to find each transformer's load as a function of the power delivered to the common load.

2.4.1 Transformers Must Have Equal Leakage Impedances

Assuming equal transformer voltage ratios, then from Fig. 2-48(c), and by applying the current-divider concept, we obtain each transformer's current as a function of the load current I_L:

$$I_{t_1} = I_L \frac{Z_{t_2}}{Z_{t_1} + Z_{t_2}} \tag{2.62}$$

and

$$I_{t_2} = I_L \frac{Z_{t_1}}{Z_{t_1} + Z_{t_2}} \tag{2.63}$$

where Z_{t_1} and Z_{t_2} are the leakage impedances of transformers 1 and 2, respectively.

From Eqs. (2.62) and (2.63), we obtain, when the leakage impedances are the same, that each transformer supplies half of the load current requirements. However, when the leakage impedances are unequal, the transformer currents are also unequal. As a result, the transformer that delivers the higher current may be overheated.

2.4.2 Transformers Must Have Equal Turns Ratios

When transformers are connected in parallel, their turns ratios must be equal. Otherwise, large no-load circulating currents will flow between the transformers.

From Fig. 2-48(b), the circulating current is

$$I_c = \frac{V_1 - V_2}{Z_{t_1} + Z_{t_2}} \tag{2.64}$$

where V_1 and V_2 are the voltages induced on the secondaries of transformers 1 and 2, respectively.

To minimize the circulating current, the "off-load" voltage tap changers should, if possible, be properly adjusted to equalize the voltages induced in the secondaries of the transformers.

If the transformers supply a load when their turns ratios are different, ordinary circuit-analysis techniques can be used to find the individual transformer currents.

2.4.3 Equal Phase Shift Between the Voltages of Paralleled Units

The three-phase units must also produce secondary line-to-line voltages in phase when they are paralleled. Otherwise, large circulating currents will flow.

For this reason, a Δ-Y transformer cannot be paralleled with a Y-Y transformer. As previously mentioned, the line-to-line voltages on the delta side of a Δ-Y transformer lead the corresponding line-to-line voltages on the star side by 30°, while the line-to-line voltages on the primary of a star-star (Y-Y) transformer are in phase with the line-to-line voltages on the secondary side.

The line-to-line voltages for one phase only of a Δ-Y and Y-Y three-phase transformer are shown in Figs. 2-49(a) and (b), respectively.

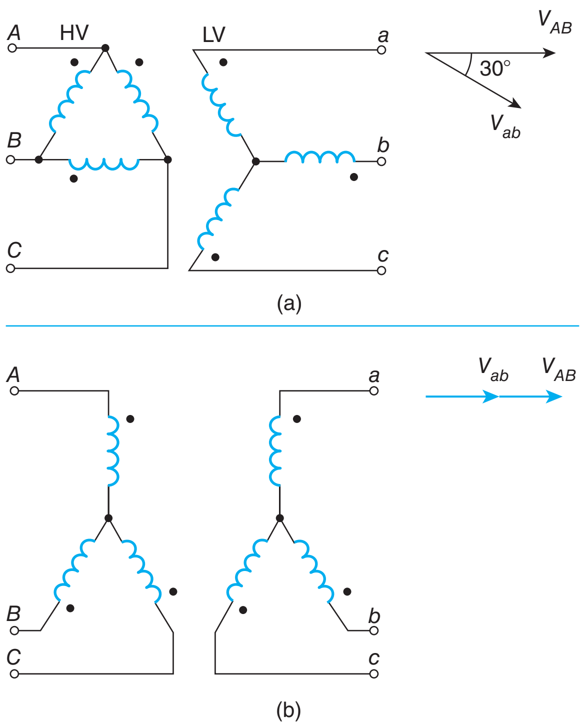

FIG. 2-49 Two-winding, three-phase transformers and a partial voltage phasor diagram. (a) Delta–star. (b) Star–star.

2.4.4 Same Phase Rotation

When two transformers are paralleled, the primary and secondary voltages must have the same phase rotation. This condition is easily verified in the field, and if necessary, the connections of the two phases can be interchanged. This will result in the same phase rotation of the three-phase voltages. When the phase rotation is not the same, large circulating currents will flow through the transformer windings, and unbalanced voltages will be delivered to the load.

The paralleling of transformers, in general, results in higher rated and short-circuit currents through the common primary feeders. This, in turn, would require more expensive primary protective devices.

Circulating Currents

To prevent large circulating currents within the windings of the paralleled transformers, the following conditions must be satisfied.

- Equal winding leakage impedances.
- Equal winding turns ratio per phase.
- Equal phase shift of the secondary voltages.

EXAMPLE 2-9 Two single-phase transformers are connected in parallel, as shown in Fig. 2-50(a), and their primaries are connected to a 25 kV supply. The data on the nameplate of each transformer are as follows:

Transformer	*Rating in kVA*	*Voltage Ratio*	*Leakage Impedances in Per-Unit*
1	600	25,000–600	$0.02 + j0.05$
2	500	25,000–610	$0.02 + j0.06$

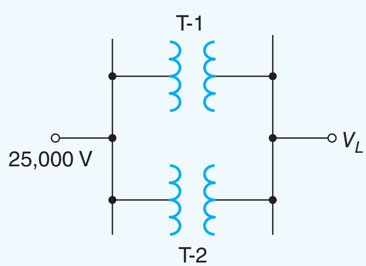

FIG. 2-50(a)

Determine:

a. The no-load circulating current.

b. The power loss due to the no-load circulating current.

SOLUTION

a. Change the per-unit values of the transformer impedances to the equivalent ohmic values of the low-voltage winding.

For Transformer 1:

$$Z_{b_L} = \frac{(600)^2}{600 \times 10^3} = 0.6\ \Omega$$

and

$$Z_{t_1} = (0.02 + j0.05)(0.6)\ \Omega$$
$$= 0.012 + j0.03\ \Omega$$

For Transformer 2:

$$Z_{bL} = \frac{(610)^2}{500 \times 10^3} = 0.74\ \Omega$$

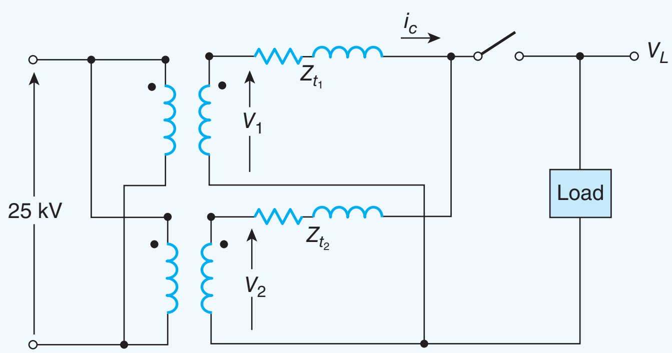

FIG. 2-50(b)

and

$$Z_{t_2} = (0.02 + j0.06)(0.74)\ \Omega$$
$$= 0.015 + j0.045\ \Omega$$

Thus, from the equivalent circuit of Fig. 2-50(b), we have

$$I_c = \frac{V}{Z} = \frac{610 - 600}{0.012 + 0.015 + j(0.03 + 0.045)} = \frac{10}{0.027 + j0.075}$$

$$= \underline{\underline{125.95\angle -70.2^\circ \text{ A}}}$$

b. The power loss due to circulating current is

$$I^2R = (125.95)^2(0.027) = \underline{\underline{426.73 \text{ W}}}$$

Exercise 2-8

Two single-phase transformers are connected in parallel, as shown in Fig. 2-51, to supply a load of 500 kVA at 480 volts and 0.90 power factor lagging. The data on the transformer's nameplate are as follows:

Transformer	*Rating in kVA*	*Voltage Ratio*	*Leakage Impedance in Percent*
1	300	4160–480	$2 + j3.5$
2	250	4160–480	$1.8 + j4.0$

For each transformer, neglect the magnetizing impedances, and determine:

a. The current.

b. The kVA loading.

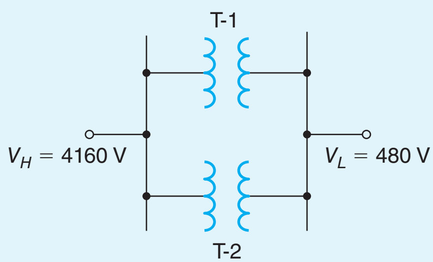

FIG. 2-51

Answer (a) $590.4\angle -23.4^\circ$ A, $452.3\angle -29^\circ$ A
(b) $283.4\angle 23.4^\circ$ kVA, $217.1\angle 29^\circ$ kVA

2.5 Instrument Transformers and Wiring Diagrams

This section discusses instrument transformers and wiring diagrams. Wiring diagrams illustrate how the instrument transformers are interconnected to the various measuring and indicating meters. These topics not only supplement the basic transformer concepts but also give an overview of the practicing engineer's work.

2.5.1 Instrument Transformers

Instrument transformers are used to measure and control electrical parameters (voltage and currents). There are many different classes, each with a specific range of measuring accuracy.

Instrument transformers used for relaying and indicating purposes have a larger margin of error than those used for measuring the power and energy on which the billing demand of a particular plant is based.

There are two types of instrument transformers: potential transformers, used for measuring voltage, and current transformers, used for measuring current (see Fig. 2-52). The equipment connected to their secondaries constitutes the so-called load, or burden. In normal operation, the burden of the current transformer is of very low impedance, while the load on the potential transformer is of very high impedance. An elementary representation of instrument transformers and some of the meters connected to their secondaries is shown in Fig. 2-53.

The polarity and the grounding of the potential and current transformers are as shown in the figure. Correct polarity identification will lead to correct instrument connections; proper device grounding will ensure the safety of personnel and the protection of equipment.

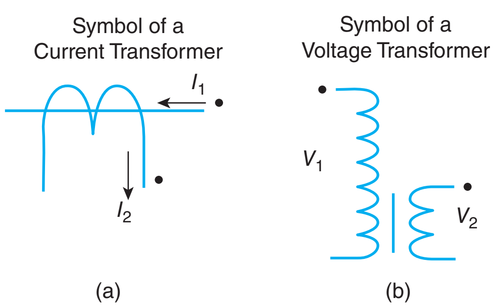

FIG. 2-52 Symbols of instrument transformers. (a) Current transformer. (b) Voltage transformer.

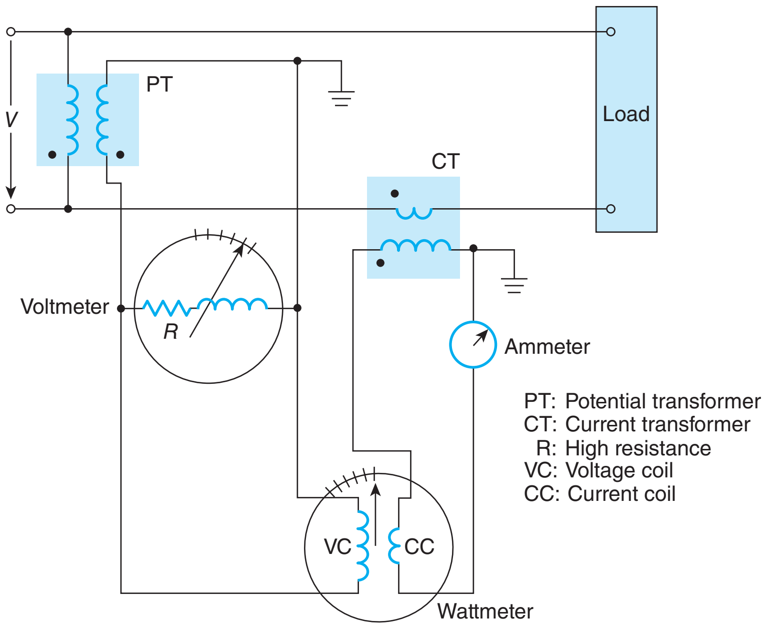

FIG. 2-53 An elementary representation of instrument transformers and measuring meters.

Potential Transformers

Potential transformers are used for stepping down voltages to nonhazardous and conventional voltage levels—normally 120 volts—for measuring and controlling purposes.

As shown in Fig. 2-54, their primary winding is connected in parallel to the high-voltage circuit whose voltage is to be measured. The voltage across the secondary winding is applied to potential coils of meters, relays, and other instruments, depending on what is desired.

Refer to Fig. 2-54(c). By applying KVL, we obtain

$$\text{input mmf} = N_c I_c + N_2 I_2 \tag{2.65}$$

where $N_c I_c$ is the mmf within the material and on whose magnitude the magnetization level of the PT depends. For a PT, the above expression becomes

$$I_p N_1 = N_c I_c + N_2 I_2 \tag{2.66}$$

where I_p is the current through the PT's primary winding. The magnitude of the primary voltage of a potential transformer is nearly constant; therefore, the magnitude of the flux within the magnetic material is also constant (see Eq. (2.7)). For

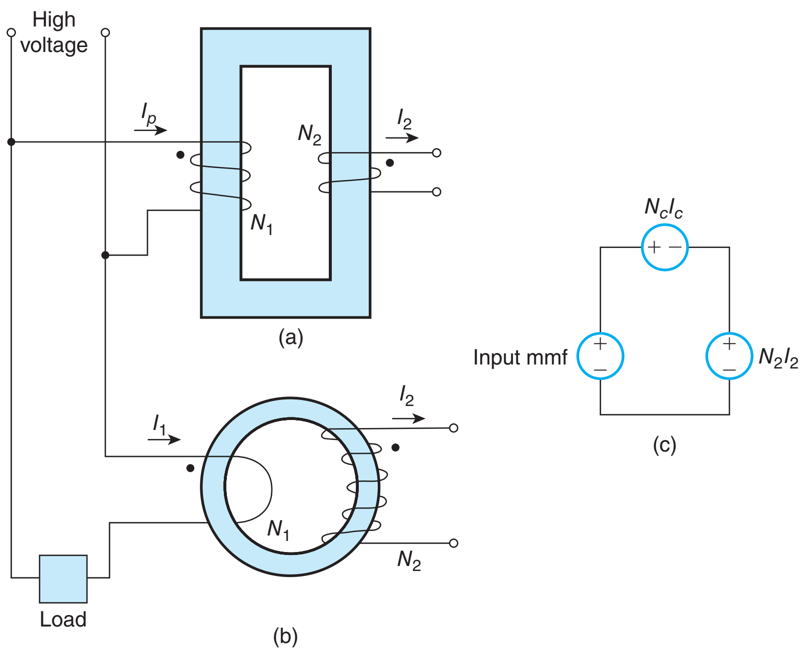

FIG. 2-54 Instrument transformers. (a) Schematic diagram for a PT. (b) Schematic diagram for a CT. (c) Magnetic equivalent circuit for instrument transformers.

all practical purposes, this results in a constant-magnitude mmf within the material, regardless of the magnitude of the load current.

The variation of the PT's magnetization level from a no-load ($I_2 = 0$) to a full-load operating condition is shown in Fig. 2-55(a). The small change in the levels of

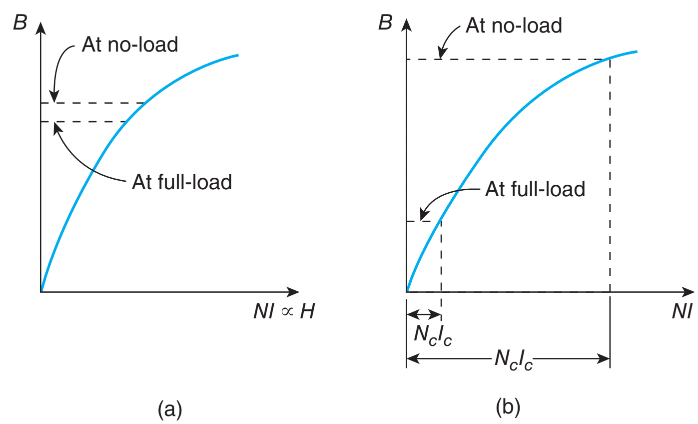

FIG. 2-55 Levels of magnetization from no-load to full-load for instrument transformers. (a) Potential transformers. (b) Current transformer.

magnetization (from no-load to full-load operating condition) is due to leakage flux that saturates part of the magnetic material.

When a PT's secondary winding is shorted, the resulting high current will produce high winding losses. Consequently, if the transformer is not properly protected, it will be damaged and may become a fire hazard.

Current Transformers

Current transformers step down the current of the primary circuit to conventional standard levels (normally 5 amperes) for measuring and controlling purposes. As shown in Fig. 2-54(b), their primary is connected in series with the circuit whose current is to be measured. The current of the secondary winding is applied to current coils of meters and relays. Under normal operating conditions, the current through the load is constant, and thus the input mmf $N_1 I_1$ to a CT is also constant.

Rewriting KVL in the magnetic circuit of Fig. 2-54(c), we have

$$\text{input mmf} = N_c I_c + N_2 I_2 \qquad \textbf{(2.67)}$$

The above expression for a CT becomes

$$N_1 I_1 = N_c I_c + N_2 I_2 \qquad \textbf{(2.68)}$$

From the last equation it becomes evident that a CT's magnetizing mmf ($N_c I_c$) varies from its maximum value at zero secondary current (open secondary) to its minimum value at maximum secondary current (shorted secondary). The variation of the CT's magnetization level from the no-load to full-load operating condition is shown in Fig. 2-55(b).

A low magnetic potential drop within the material corresponds to low levels of flux and core loss. For this reason, energized CT's have their secondaries either shorted or connected to low impedance devices. When an energized CT has its secondary open-circuited, it will be damaged (a problem usually encountered in the commissioning of new plants) by excessive magnetic losses.

A CT hooked up to a live feeder should never have its secondary open-circuited. This would result in a higher level of magnetization, which in turn would induce a high potential across the secondary terminals. This will endanger working personnel and will destroy the CT (owing to excessive core loss).

For the purposes of comparison, the characteristics of CT's and PT's are summarized in Table 2-3.

TABLE 2-3 Main characteristics of PT's and CT's

Description	*PT*	*CT*
Input winding connection, with respect to plant's load	Parallel	Series
Step down the magnitude of the voltage	Yes	
Step down the magnitude of the current		Yes
Levels of magnetization depends on	Supply line voltage	Current through its secondary
Damaged when its secondary is	Short-circuited	Open-circuited
Cause of damage	High copper loss	High core loss

Exercise 2-9

When the single-phase load of Fig. 2-54 is accidentally shorted, what will be the effect on the potential and current transformers?

Optical Instruments

Optical instrument transformers refer to potential and current transformers, which contrary to classical PT's, measure the voltage and current to a distribution system without being in contact with the live conductors. They are very expensive and are presently used in high-voltage substations. Their usage, however, will be extended to lower voltages because of their great advantages, such as security of personnel, very low maintenance, high accuracy, no magnetic or copper losses, prevention of fires, no ferroresonance, and small size.

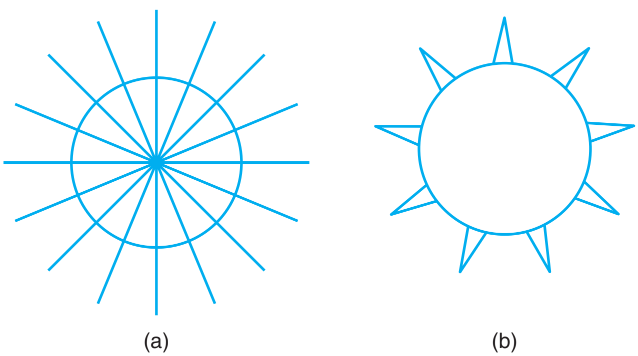

FIG. 2-56 Light bulb. (a) Unpolarized light. (b) Light (polarized) as seen with reading glasses.

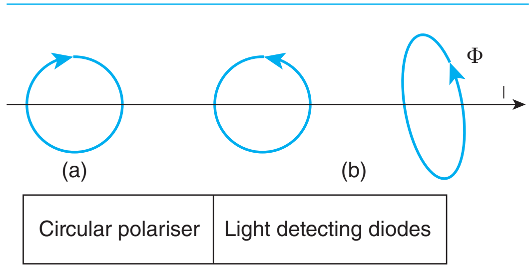

FIG. 2-57 Schematic diagram of an optical CT. (a) Polarized circular light encircling a wire and magnetic field of cable's current. (b) Generation and detection of polarized light.

*In general, a light polarizing device, depending on its configuration, transmits part of the light. This process of polarization can be easily observed at night when you use reading glasses and look at the street incandescent lights and/or when you look toward the lights of incoming cars when driving (Fig. 2-56).

Optical Current Transformers

The optical CT consists of two sections, one of which produces opposite rotating circular polarized lights and the other incorporates light detecting diodes.

For a given wire to a distribution network, two circular polarized lights are generated (Fig. 2-57), one of which is aided by the magnetic field of the conductor's current ($\phi \; \alpha \; I$) and the other is opposed. The change in the illumination level is detected by the light-detecting diodes. The diode's current is further processed, and the output of the optical sensor is transmitted to the control room by wireless technology or by optic fibers.

PT's

The sensor of an optical PT detects the intensity of the electric field (E) produced by the conductor's current. The intensity depends on the distance of the sensor from the section of the wire and the voltage of the wire's field.

The corresponding voltage produced ($v = -\int E \, dx$) is the weighted average of the output of the detecting elements.

Some manufacturers combine the optical CT's and PT's in the same unit.

2.5.2 Wiring Diagrams

Wiring diagrams are electrical drawings that use conventional symbols to indicate the actual hook-up of meters. The wiring diagram of a digital meter is shown in Fig. 2-58. It measures many parameters such as voltage, current, power, power factor, and harmonics. Such units are equipped with a display and a set of push buttons. When a parameter is to be displayed, a corresponding push button is depressed. These meters can also send "measuring pulses" to computers, which in turn can print out the instantaneous variations of the system parameters.

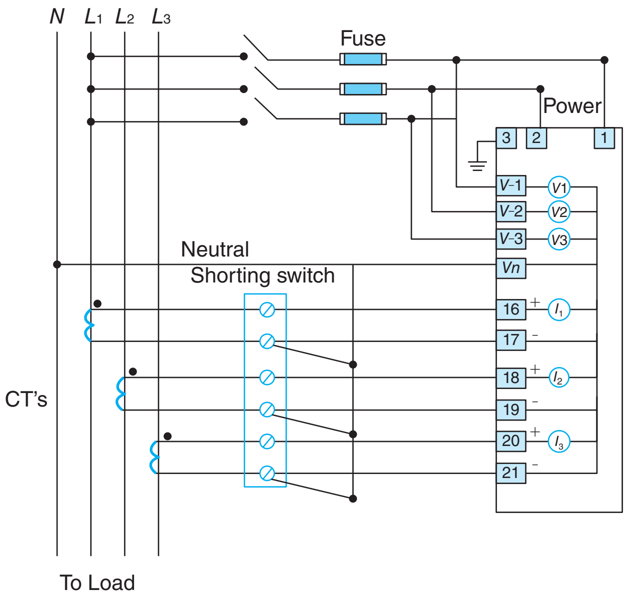

FIG. 2-58 Wiring diagram of a digital multimeter.

EXAMPLE 2-10

For the single-phase transformer shown in the one-line diagram in Fig. 2-59, determine the ammeter reading for the following operating conditions:

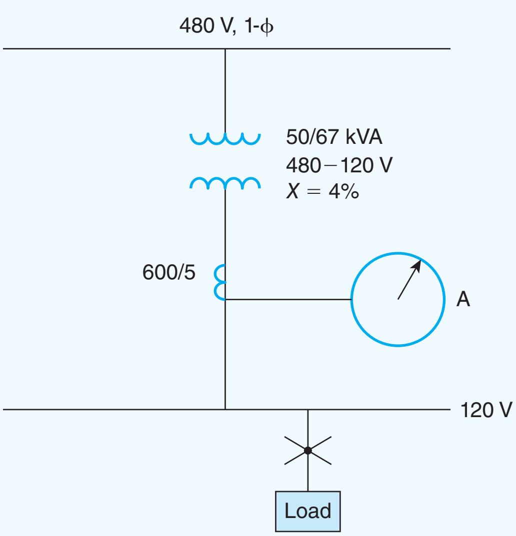

FIG. 2-59

a. The transformer delivers 80% of its base kVA at nominal voltage.
b. The transformer delivers 100% of its fan-rated kVA.
c. The transformer's secondary feeder is shorted.

SOLUTION

a. When the transformer delivers 80% of its rated kVA, the magnitude of the current in the line (or the ammeter reading) is

$$I = 0.8\left(\frac{50 \times 10^3}{120}\right) = \underline{\underline{333.33\text{ A}}}$$

The CT's secondary current will be

$$I_{CT} = I_{\text{actual}} \times \text{turns ratio}$$
$$= 333.33\,\frac{5}{600} = 2.78\text{ A}$$

This is also the current through the ammeter, but the ammeter is always calibrated to give the actual current through the load.

b. Similarly, the ammeter reading will be

$$I = \left(\frac{67 \times 10^3}{120}\right) = \underline{\underline{558.33\text{ A}}}$$

The CT's secondary current will be

$$I_{CT} = 558.33\left(\frac{5}{600}\right) = 4.65\text{ A}$$

c. The impedance of the transformer depends on its base kVA and not on its fan-rated kVA. The kVA's significance is that if the transformer windings are externally cooled, they can carry higher currents without becoming overheated. Since the applied voltage and the transformer impedance remain constant, the short-circuit current is independent of the level of operating kVA. Thus, the ammeter reading on short-circuit conditions is

$$I = \left(\frac{1}{0.04}\right)\frac{50 \times 10^3}{120} = \underline{\underline{10.42\text{ kA}}}$$

The current through the CT's secondary winding will be

$$I = 10.42 \times 10^3\,\frac{5}{600} = 86.81\text{ A}$$

Example 2-11

a. Refer to Fig. 2-59. When the ammeter reads 1500 A and has an accuracy of 99%, what is the apparent power drawn by the inductive load?
b. What is the power factor of the load when the kW meter reads 2000 kW and the reactive power meter reads 1500 kVAR?

SOLUTION

a. The scales on switchboard-type meters are calibrated in terms of the line-side parameters. Thus, the current through the 4.16 kV lines is

$$I_L = \frac{1}{0.99}(1500) = 1515.15 \text{ A}$$

By definition, the apparent power is

$$|S| = \sqrt{3}V_{L\text{-}L}I_L = \sqrt{3}(4.16)(1515.15) = \underline{\underline{10{,}917.17 \text{ kVA}}}$$

b. The real and reactive powers are given, respectively, by

$$P = \sqrt{3}V_{L\text{-}L}I_L \cos\theta$$
$$Q = \sqrt{3}V_{L\text{-}L}I_L \sin\theta$$

Thus,

$$\theta = \arctan\frac{Q}{P} = \arctan\frac{1500}{2000} = 36.9°$$

The power factor is

$$\cos 36.9° = \underline{\underline{0.80 \text{ lagging}}}$$

Exercise 2-10

a. Why is the secondary of a PT normally supplied with a fuse but not the secondary of the CT?
b. Explain why a voltmeter never measures the voltage and an ammeter never measures the current.

Ferroresonance

Human Resonance

Resonance in general is a condition that results in maximum response (love, hate, indifference, and so forth). Some people instinctively love red colors, while others hate them or yet some others are indifferent to them. The future of an individual most likely will be strongly influenced by his or her inner inclination (resonance) in conjunction with that of fellow humans (boss, partner, friend, and such). It has been reported that an international bank, in order to select the most suitable employees to a managerial position, subjects all candidates to a questionnaire (searching for resonance) and from this, a decision is made that is apparently 100% accurate.

Linear Resonance

In the linear resonance, the overvoltages and overcurrents are of predicable magnitude. Such a resonance takes place when the circuit's inductive reactance is equal to the capacitive reactance. Furthermore, the circuit's power factor is equal to 100%, and the magnitude of the resulting overvoltages and overcurrents are sinusoidal and of predicable frequency. Similarly, a building is characterized with linear resonance whose frequency is in the range of 4–6 Hz.

Nonlinear Resonance

In nonlinear resonance the voltages and current are very high and of unpredictable frequency. The nonlinear resonance is known as a ferroresonance. It was observed almost 100 years ago that ferroresonance was taking place in distribution systems whose transformers were made from ferrous materials (iron laminations).

Ferroresonance depends on the circuit's initial conditions, the applied voltage, the nonlinear circuit inductance, and on the capacitance between cables and/or cables and ground. In ungrounded systems, the voltages developed could be up to 4 pu, while in the grounded systems, voltages could reach 2.5 pu.

The inductance is inversely proportional to the permeability, which in turn is the slope of the material's B-H diagram. At higher than nominal voltages, the transformer is driven into saturation, the permeability is reduced, and thus the inductance is increased.

Many causes can initiate ferroresonance, but the main one is the variable inductive reactance of the power, potential, and measuring potential transformers that are driven into saturation. The initiation of this phenomenon could be attributed to the lightning strokes, breaker switching, or malfunctioning and single-phase operation of a three-phase distribution system.

One way to mitigate the effects of resonance is to oversize the transformers so that an accidental increase in the supply voltage will not drive them into saturation and/or add a resistance (damping effect) on the secondary windings of the transformers.

It has been well publicized that if a small step-down PT and a small capacitor in a control system initiated ferroresonance, that can cause the failure (insulation damage) of dozens of downstream motors.

K-Factor of Transformers

Introduction

A transformer designated as *k*-type (*k*-4, *k*-9, *k*-13, etc.) indicates that it will not be overheated when the *k*-factor of the line currents is lower than that of the transformer.

When $k = 1$, it means that the transformer will not be overheated when it supplies rated power to linear load—that is, when the load does not draw any harmonic current. The rate of heat loss within the transformer is the sum of the core loss (eddy-current and hysteresis loss) plus the stray losses and I^2R of its windings.

The eddy-current losses and hysteresis losses are nonlinear functions of the currents' frequency.

By far, the largest effects of the harmonic components of the currents are on the eddy-current losses.

Calculation of the *K*-Factor

The *k*-factor is defined as follows:

$$k = \sum_{h=1}^{h_{max}} I_h^2 h^2 \qquad (2.69)$$

where h stands for the number of the harmonic and I_h is the rms current of harmonic h, in per unit of the line current.

EXAMPLE 2-12

The line current (ι) in a power distribution system is

$$\iota = 10 \sin \omega t + 4 \sin 3\, \omega t + 2 \sin 5\, \omega t \ldots.$$

Determine the *k*-factor.

Solution

The rms value (I) of the line current is

$$I = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 7.746 \text{ A}$$

and the per-unit values of the current components are

$$I_1 = \left(\frac{10/\sqrt{2}}{7.746}\right) = 0.9129$$

$$I_3 = \left(\frac{4/\sqrt{2}}{7.746}\right) = 0.3651 \text{ pu}$$

and

$$I_5 = \left(\frac{2/\sqrt{2}}{7.746}\right) = 0.1826 \text{ pu}$$

From Eq. (2.69), we obtain

$$k = [(0.9129)(1)]^2 + [(0.3651)(3)]^2 + [(0.1827)(5)]^2$$
$$= \underline{\underline{2.8664}}$$

2.6 Transformer's Nameplate Data

Some of the essential markings on a transformer's nameplate are as follows.

Apparent Power Rating in kVA

This parameter indicates the designed kVA base transformation capability of the transformer. As long as the transformer operates at or below its marked kVA, it will not overheat. This is also the base power for the per-unit calculations of the transformer impedances. When transformers are equipped with cooling fans, they can deliver higher kVA than their rated value without being overheated.

For example, a transformer designated as

1500/2000 kVA

has a base transformation capacity of 1500 kVA. However, with the use of cooling fans, the transformer can deliver 2000 kVA (33% increase) without being overheated. When a second set of fans is used, the transformer can deliver an additional 25% kVA without being overheated. Also, when a special winding insulation is used, the transformer can safely deliver an additional 12% kVA. For example, a transformer with nameplate data of 12/16/20/22.4 MVA should be interpreted as follows:

12 MVA	is the base power
16 MVA	is its capacity with one set of fans (1.33)
20 MVA	is its capacity with two sets of fans (1.25)
22.4 MVA	is its capacity due to upgraded winding insulation (1.12)

The ohmic value of the transformer's impedance remains the same, regardless of whether the transformer delivers its base or its fan-rated kVA. The resistance of the windings depends on the temperature of the conductors and will change somewhat with increases or decreases in load current.

Impedance in Percent

This is the magnitude of the equivalent winding or leakage impedance. The transformer's magnetizing impedance is not included on the nameplate data. The winding impedance (expressed in percent) indirectly gives the equivalent transformer's impedance *I* ohms and also indicates what percent of the rated voltage is required to circulate rated current through the windings of the transformer when the secondary is shorted.

The leakage impedance varies with the size of the transformer, class voltage, type of winding (copper or aluminium), and frequency. For the standard voltages and in the range of 1000 to 2000 kVA, the transformer's impedance is about 6%.

Efficiency in Percent

This is the efficiency of the transformer when it delivers rated kVA at unity power factor. Generally, the higher the kVA capacity of the transformer, the higher its efficiency. In the range of 100 to 2000 kVA, the efficiency is usually between 96 and 99%.

"OFF-Load" Voltage Tap Changers

Normally, transformers are equipped with four "OFF-load" voltage tap changers ($\pm$ 2.5% and $\pm$ 5%). The positive tap settings are used to compensate for voltage drop through the transformer's upstream feeders, while the negative tap settings are used to compensate for generated voltages that are slightly above nominal values.

Winding Connection—Rated Voltages

The nameplates of three-phase transformers also indicate whether their windings have a delta (Δ) or a star (Y) connection and give their corresponding rated voltages. The latter may be used in establishing the line-to-line or the phase-to-neutral turns ratio. Nameplate data also indicate whether provisions have been made for grounding the neutral terminal of the star-connected windings.

2.7 Conclusion

Transformers change the voltage and the current from one level to another and, as such, constitute the main interconnecting link between two systems of different voltages.

The design, operation, and analysis of all types of transformers are based on the induction principle, which says that a voltage is induced in coils whose flux linkages ($N\phi$) change as a function of time. Under ideal conditions, the induced voltage and the resulting current in a secondary coil are related to the corresponding parameters of the source coil (primary coil) by the windings turns ratio.

Each type of transformer can be analyzed by using either the magnetic or the electric equivalent circuits. Although using the magnetic circuit is simpler, the trend throughout the industry is to employ the electric equivalent circuits.

The core and copper losses of transformers—even though they are a small fraction of the power-transformation capability of any transformer—often constitute the criteria for selecting transformers. This is because the purchasing cost of a transformer is much lower than the cost of the energy losses within the transformer over its operating life. (A kilowatt consumed continuously over a 20-year period has a present worth of about $7215 when the annual interest rate is 6% and the cost of energy is 15¢/kWh and increases 4% per year. In comparison, the cost of a 25-kVA transformer that has 1 kW of power loss is about $2000.)

In the analysis of circuits that include transformers, it is much more convenient to use per unit values because the impedances, voltages, or currents, when expressed in per-unit, are the same in either side of the transformer.

Throughout the various sections of this chapter, the short-circuit MVA of transformers is calculated because of its importance in equipment selection (circuit breakers, fuses, etc.). When properly selected, this equipment safeguards personnel and downstream apparatuses.

Single-phase power transformers have relatively small ratings, usually in the range of 1–100 kVA.

The coils of the transformer are magnetically coupled, and the relative polarity of the induced voltages is normally identified by a dot.

By convention, the dotted terminal is at higher potential than the undotted terminal. This leads to determination of the direction of current flow and flux. This is of extreme practical importance for proper field connections of measuring equipment and of protective relays. Misinterpretation of the dotted terminals will result in incorrect meter indication, a problem that usually arises during the commissioning of industrial plants.

The analysis of *three-phase*, two-winding transformers is simplified by using their electric or magnetic per-phase equivalent circuits. An additional simplification results when the per-unit values of the transformer parameters are used. The latter remove the difficulty of long calculations and the confusion that may arise from the various three-phase transformer connections.

Transformers of 5–20 MVA rating have a nominal impedance in the range of 5% to 8% and an efficiency of 0.97 to 0.99.

The phase shift between the primary and secondary line-to-line voltages depends on the transformer's connections. For the delta–star connection, the voltages on the HV side lead their corresponding LV side by 30 degrees. The majority of three-phase transformers are of Δ-Y type. They tend to maintain sinusoidal secondary voltages, and the secondary star windings can be used to ground the distribution network and to connect line-to-neutral loads.

Three-phase transformers are normally equipped with a set of ventilating fans (the additional cost is 5% above nominal). As a result, the transformer's transformation capacity is increased by 33%.

Autotransformers are used in large power systems and in testing. Standard transformers are paralleled in order to increase the reliability of a substation (two smaller units are more reliable than a larger unit) and to provide additional transformed kVA that meets a plant's increased demand. In order to parallel two transformers, they must have equal impedances, equal turns ratios, the same phase rotations, and equal phase shifts between the primary and secondary open-circuited voltages.

Instrument transformers step down the voltage and the current to more standard and less dangerous levels.

Usually, the secondary of a potential transformer is rated at 120 V, while the secondary of a current transformer is rated at 5 A.

The burden, or the load (voltmeters, voltage coils of kW-meters, etc.), of potential transformers is of very high impedance; conversely, the burden of the current transformers is of very low impedance.

Energized potential transformers are damaged—owing to high copper loss—when their secondary is shorted. Current transformers are damaged—owing to high core loss—when their secondary is open-circuited.

Wiring diagrams are electrical drawings that use conventional symbols to indicate the actual field connections of the measuring equipment. As such, wiring diagrams are of fundamental importance to power-distribution engineers, to those who participate in the commissioning of industrial plants, or to those whose responsibility it is to maintain the plant's electrical equipment.

In ferroresonance, high and repetitive voltages are developed within the distribution system. It results from circuit breaker switching, lightning strokes, and the like that drive the plant's coils and transformers into saturation.

The *k*-rating of the transformers reveals the level of harmonics at which it will not be overheated. The sources of harmonics are lighting fixtures, microprocessors, and variable speed drives.

2.8 Summary

The following tables summarize the main concepts on transformers and their technical characteristics and parameters. The main concepts of the chapter are condensed in Table 2-4, and typical manufacturer's data are given in Table 2-5 through Table 2-8.

TABLE 2-4 Summary of Main Transformer Concepts

Item	Description	Remarks
1	Principle of operations of transformers: $$v = -N\frac{d\phi}{dt}$$	Eq. (2.6)
2	Fundamental transformer equation: $$V = 4.44Nf\phi_m$$	Eq. (2.7)
	Ideal Transformer Relationships	
3	$$V_2 = V_1\frac{N_2}{N_1}$$	Eq. (2.11)
4	$$I_2 = I_1\frac{N_1}{N_2}$$	Eq. (2.14)
5	$$Z_1 = Z_2\left(\frac{N_1}{N_2}\right)^2$$	Eq. (2.17)
6	*Short-circuit test* $$I_z = I_{\text{rated}}$$ $$V_z = (2\% \rightarrow 12\%)V_{\text{rated}}$$ Measure I_z, V_z, and P_z. Calculate: $$R_e = \frac{P_z}{I_z^2}, \quad \lvert Z_e\rvert = \frac{V_z}{I_z}, \quad X_e = \sqrt{Z_e^2 - R_e^2}$$	Section 2.1.3
7	*Open-circuit test* $$I_{\text{exc}} = (3\% \rightarrow 10\%)I_{\text{rated}}$$ $$V_{\text{exc}} = V_{\text{rated}}$$ Measure: $$I_{\text{exc}}, V_{\text{exc}}, \text{and } P_{\text{exc}}$$ Calculate: $$R_m = \frac{P_{\text{exc}}}{I_{\text{exc}}^2}, \quad \lvert Z_m\rvert = \frac{V_{\text{exc}}}{I_{\text{exc}}}, \quad X_m = \sqrt{Z_m^2 - R_m^2}$$	Section 2.1.3
8	Transformer operates at maximum efficiency when $P_z = P_{\text{exc}}$	Eq. (2.38)
9	$$\text{Regulation\%} = \frac{\lvert V_{nl}\rvert - \lvert V_{fl}\rvert}{\lvert V_{fl}\rvert}(100)$$	Eq. (2.39)

(Continued)

TABLE 2-4 *(Continued)*

Item	Description	Remarks
10	$Z_{e_{pu}} = V_{z_{pu}}$	Eq. (2.42)
	3-ϕ Transformers	
11	$N_A I_A = N_a I_a$	Eq. (2.51)
12	Star–delta impedance: $Z_y = \dfrac{Z_\Delta}{3}$	Eq. (2.53)
13	Third harmonic component of exciting current: $i_3 = 3I_{m_3} \cos 3\omega t$	
13a	Evaluation of a transformer's k-rating: $k = \sum_{h=1}^{h_{\max}} I_h^2 h^2$	Eq. (2.69)
	Autotransformers	
14	The ampere-turns of each winding are constant, regardless of the type of transformer connection. That is, $(NI) = \text{constant}$	Eq. (2.60)
15	Neglecting losses, $(\text{kVA})_{\text{input}} = (\text{kVA})_{\text{output}}$	Eq. (2.61)
	Parallel Operation of Transformers	
16	To parallel two transformers, the following conditions must be satisfied: a. equal impedances b. equal turns ratios c. equal phase shift between the primary and the secondary open-circuited voltages d. the same phase rotations	Section 2.4
	PT's and CT's	
17	When short-circuited, energized potential transformers are damaged because of high copper loss	Section 2.5.1
18	When open-circuited, energized current transformers are damaged because of high core loss	Section 2.5.1

TABLE 2-5 Typical parameters of dry-type transformers with copper windings at 60 Hz

a. Single phase (480–240/120 V)

	Losses in Watts at 170°C		Efficiency in Percent at 170°C		
kVA	Core	Copper	Full-Load	¼ Full-Load	Impedance in Percent at 170°C
10	70	275	96.7	96.6	3.6
25	110	825	96.4	97.5	4.1
50	150	2000	95.9	97.8	5.7
100	280	3350	98.5	98.1	4.7

b. Three-phase (480–208/120 V)

15	130	410	96.5	96.0	3.6
75	300	3100	95.7	97.4	5.7
150	540	5000	96.4	97.8	4.7
500	1120	11,531	97.5	98.5	5.0

Based on data from Westinghouse Canada Inc.

TABLE 2-6 Parameters of 4160–480/277 V, dry-type transformers with copper windings

	Resistance in Ohms at 170°C			Losses in kW			
kVA	HV	LV	Exciting Current in Percent	Core	Copper	Efficiency at Full-Load	Impedance in Percent
500	2.9	0.011	1.5	1.2	7.6	98.3	5.5
1000	1.09	0.004	1.12	3.8	13.1	98.3	5.9
1500	0.75	0.003	0.70	4.0	20	98.4	6.1
2000	0.48	0.016	0.65	4.8	23	98.6	6.5

Based on data from Westinghouse Canada Inc.

TABLE 2-7 Parameters of a 15/20/22.4 MVA, 60–4.16/2.4 kV, liquid-type transformer with copper windings

Exciting Current in Percent	Core Loss in kW	Copper Loss in kW	Impedance in Percent
0.41	15.0	65	7.0

Based on data from Westinghouse Canada Inc.

TABLE 2-8 Parameters of 4160–480/277 V, 60 Hz, dry-type transformer with copper and/or aluminum windings

	Cost in 1988		Core Loss in kW at 170°C		Copper Loss in kW at 170°C		Total Weight in kg		Estimated Impedance in Percent
	Windings		Windings		Windings				
kVA	Copper	Aluminum	Copper	Aluminum	Copper	Aluminum	Copper	Aluminum	
750	$22,800	$19,000	3.0	3.4	8.1	8.8	2100	2050	6
1000	26,400	22,000	3.8	4.0	13.1	14.2	2500	2425	6
1500	30,600	25,000	4.2	4.2	20	21.5	3300	3100	6

Based on data from Westinghouse Canada Inc.

2.9 Review Questions

1. In general, what fraction of the transformer's output power is its iron and winding losses?
2. Show that 1 kW consumed continuously over a 20-year period has a present-worth value of $7215. Assume the interest rate is 6% per year, the cost of energy is 5 cents/kWH, and the increase in the cost of energy is 4% per year.
3. Explain why the magnetizing impedance is not a constant parameter but depends on the level of magnetization.
4. How would you measure the magnetic potential drop $(N_c I_c)$ in a magnetic material? How is this potential related to the permeability of the material?
5. Explain why the per-phase analysis of a three-phase balanced system is analogous to the per-pole analysis of a magnetic circuit.
6. For maximum efficiency, how is winding loss related to core loss?
7. Why does the selection of the transformer's primary protective devices depend on the time-current waveform of the inrush current?
8. Sinusoidal voltages produce sinusoidal fluxes, which in turn produce nonsinusoidal exciting currents. Explain.

9. What is the predominant harmonic of the exciting current and of the inrush current of a transformer?

10. Show that the per-unit value of the transformer's leakage impedance is the same whether it is referred to as the HV or the LV winding.

11. Draw the power triangle for a balanced three-phase inductive load and give the formulas for real, imaginary, and complex power.

12. Nonsinusoidal exciting currents produce sinusoidal secondary voltages. What are the two essential requirements for the existence of nonsinusoidal exciting currents?

13. Explain why the most important characteristic of an autotransformer is that the current in its windings is the same as when the transformer operates as a conventional two-winding transformer.

14. What four conditions must be satisfied before any two transformers are paralleled?

15. What is the essential difference between a CT and a PT?

16. Explain the nameplate data marked on your laboratory's transformer.

17. Compare the following: exciting current, inrush current, load component of primary current and short circuit current of a transformer.

2.10 Problems

2-1 A 50 kVA, 2300–230 V, 60 Hz, single-phase step-down transformer has primary and secondary leakage impedances of $(0.5 + j2.6)$ and $(0.005 + j0.026)$ ohms, respectively. Neglecting the magnetizing impedance, determine:

a. The equivalent circuit, as seen from the high-voltage and the low-voltage windings.

b. The secondary voltage, when the transformer is connected to a 2300 V source and delivers rated current to a load at 0.9 Pf lagging.

2-2 Data from the open- and short-circuit test on a 10 kVA, 2400–240 V, 60 Hz step-down transformer are as follows:

Test	Voltage in Volts	Current in Amperes	Power in Watts
Open circuit	2400	0.35	150
Short circuit	12	41.67	320

Determine:

a. The equivalent circuit, as seen from the high- and low-voltage winding.

b. The primary voltage, when the transformer delivers rated current at 0.9 power factor to a 240 V inductive load.

c. The voltage regulation and efficiency.

d. The per-unit value of the transformer's equivalent leakage impedance.

2-3 A single-phase 50 kVA, 2400–120 V, 60 Hz transformer has a leakage impedance of $(0.023 + j0.05)$ per-unit and a core loss of 600 watts at rated voltage. Determine:

a. The load current for maximum efficiency.

b. The efficiency for the condition in (a) and the efficiency when the transformer delivers rated current. In both cases, assume unity power factor.

c. The power factor of the load that will result in the best regulation (0%). Assume that the load current and the primary voltage remain constant at their rated values.

2-4 A 100 kVA, 480–120 V, 60 Hz, single-phase transformer has an efficiency of 95.75% at rated conditions and unity power factor. Its leakage impedance is (2.5 + j5.0) percent. The transformer's open-circuit test data are as follows:

Voltage in Volts	*Power in Watts*	*Frequency in Hertz*
100	1400	50

a. Determine the eddy-current and hysteresis losses at 60 Hz and 120 V.

b. Determine the rate at which heat is produced within the structure of the transformer at 100 V and 50 Hz due to the friction caused by the motion of the magnetic domains.

c. Can the transformer be used to deliver its rated power at rated voltage and 50 Hz? Explain.

2-5 A 15 kVA, 480–240/120 V, 60 Hz, single-phase transformer has equal primary and secondary leakage impedances of (0.01 + j0.02) per unit. Determine:

a. The no-load voltage across the secondary terminals at rated flux.

b. The voltage regulation when the transformer is connected to a 480 V source and delivers rated current at a power factor of:

1. Unity.
2. 0.80 leading.
3. 0.80 lagging.

2-6 A transformer has a leakage impedance of (0.015 + j0.06) per unit. When the transformer delivers rated current at 0.9 Pf lagging, its efficiency is 97%. Determine:

a. The nominal core loss in per-unit and the voltage regulation.

b. The no-load voltage tap setting that will minimize the voltage regulation.

2-7 Figure P2-7 shows a core-type 150 kVA, 4160–480/277 V, three-phase transformer. Connect the primary in delta and the secondary in star. The neutral is to be solidly grounded. Show coil polarities and direction of flux lines and find the coil currents for the following load conditions:

a. A load drawing 60 A is connected from line a to neutral.

b. A load drawing 60 A is connected from line a to line b.

c. A 3-ϕ delta-connected load of 100 kW, 480 V, 0.90 efficient, and 0.8 power factor lagging is connected across the secondary.

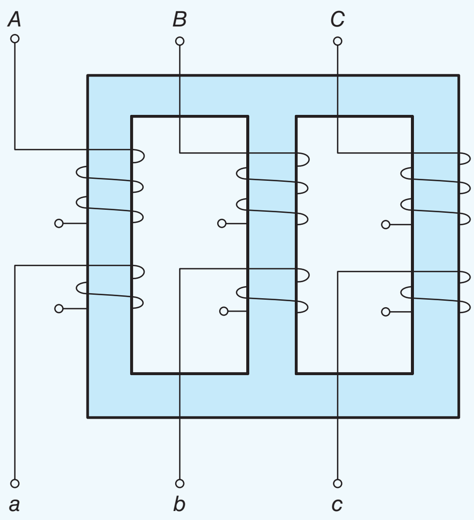

FIG. P2-7

2-8 A three-phase, 2000 kVA, 25,000–480/277 V, Δ-Y transformer supplies two three-phase loads through feeder B as shown in Fig. P2-8. The Δ-connected high-voltage transformer windings are connected

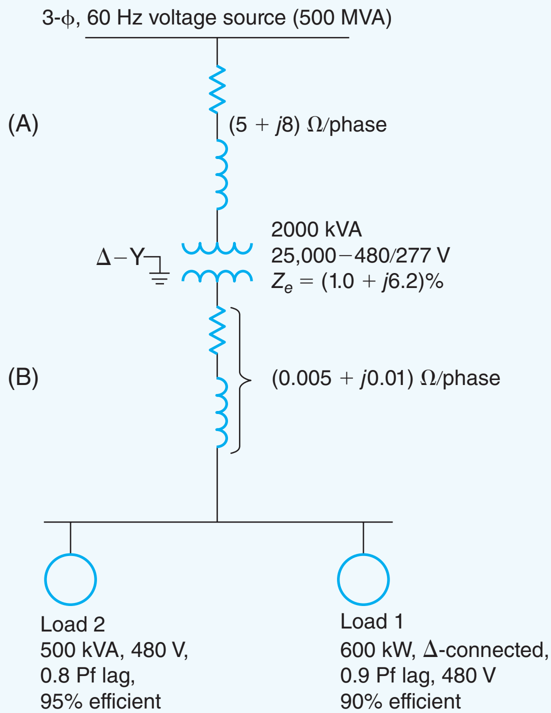

FIG. P2-8

through cable A to a 3-ϕ, 60 Hz power source whose short-circuit MVA is 500. The ratings of various equipment are as shown in the diagram. If the three-phase loads draw rated currents at rated voltage, determine:

a. The per-unit values of the impedances of cables A and B, using the ratings of the transformer as base values.

b. The total losses of the transmission system.

c. The voltage required at the source.

2-9 Figure P2-9 shows a three-phase, five-wire (3-ϕ, 5W) distribution system. If the connected loads draw rated currents, determine:

a. The minimum required kVA capacity of the transformer.

b. The phase currents through each of the six transformer coils.

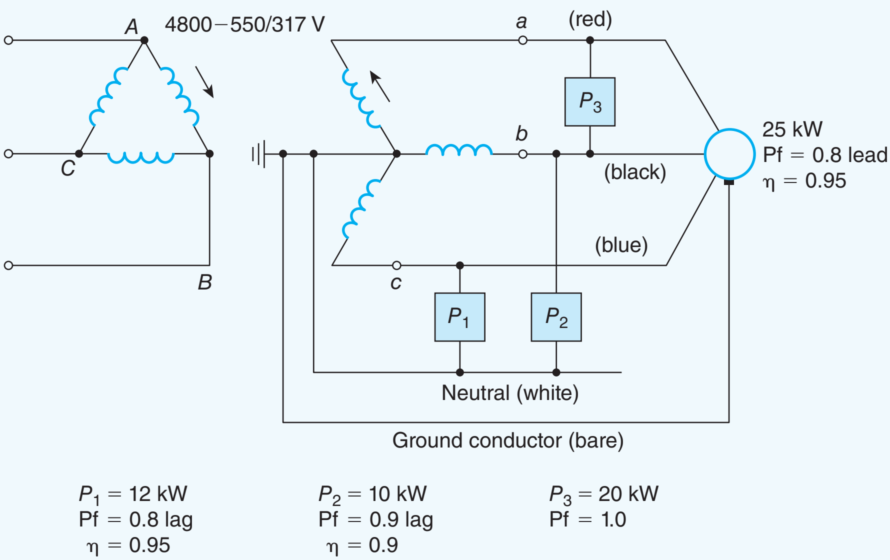

FIG. P2-9

2-10 Figure P2-10 shows a three-phase transformer whose neutral point is grounded through a 1.0 ohm resistor. The transformer delivers rated voltage to the three-phase heater. When the incoming live part of cable *C* (phase *ABC*) accidentally makes contact with the grounded metallic enclosure of the heater, estimate (assuming negligible transformer leakage impedance) the following:

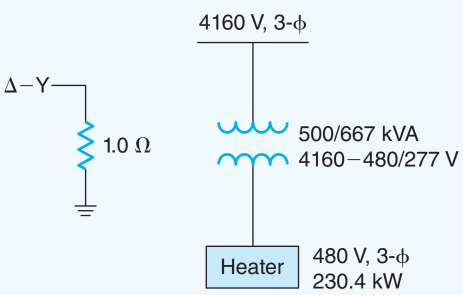

FIG. P2-10

a. The line and ground currents.

The voltage from the heater's enclosure to ground.

The reaction of the upstream protective devices.

b. Repeat (a), assuming that the enclosure is not grounded.

c. Repeat (a), assuming that the neutral of the transformer is solidly grounded.

d. Repeat (a), assuming that the neutral of the transformer is not solidly grounded.

2-11 A 5 kVA, 480–120 V, two-winding transformer is to be connected as an autotransformer to supply a 480 V load from a 600 V supply. Determine:

a. The autotransformer kVA.

b. The untransformed kVA delivered to the load.

2-12 A two-winding, 2400–1000 V, 200 kVA transformer with a leakage impedance of $(0.02 + j0.06)$ per unit has an efficiency of 0.96 when it delivers rated current at 0.85 power factor lagging. This transformer is to be connected as an autotransformer in order to supply a 2400 V load from a 3400 V supply. Determine:

a. The kVA rating of the autotransformer. What percentage of this kVA passes through the transformer unaltered?

b. The efficiency at full load and at 0.85 power factor.

c. The primary current when the secondary of the transformer is shorted.

2-13 Two single-phase transformers are connected in parallel, as shown in Fig. P2-13, to supply rated voltage and current to a 1000 kVA, 480 V, 0.95 power-factor-lagging load. The nameplate information for each transformer is as follows:

Transformer	*Rating in kVA*	*Voltage Ratio*	*Leakage Impedances in Per-Unit*
1	550	4160–480	$(0.02 + j0.04)$
2	550	4160–468	$(0.02 + j0.05)$

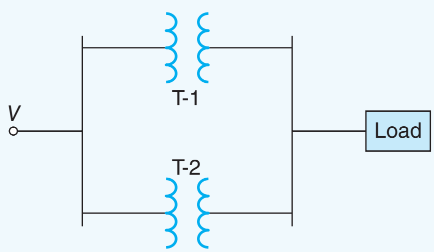

FIG. P2-13

Assuming constant primary voltage and negligible magnetizing current, determine:

a. The primary voltage.

b. The current through each transformer.

c. The output kVA of each transformer.

d. The no-load circulating current.

e. The current in the common primary feeder if the load is shorted.

2-14 A transformer is connected to a 25 kV substation through a cable, as shown in the one-line diagram in Fig. P2-14. The characteristics and parameters of the system's components are as shown.

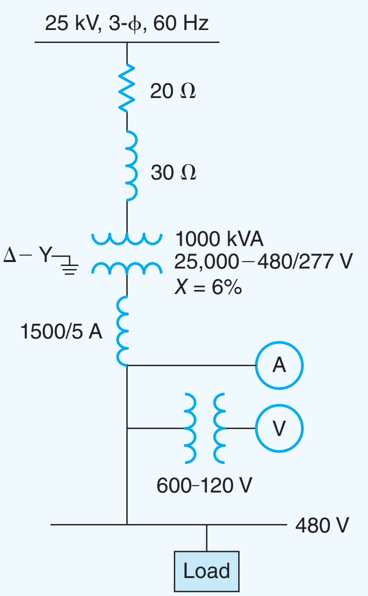

FIG. P2-14

a. Determine the voltmeter indication, the current through the secondary of the CT, and the ammeter indication for the following operating conditions:

1. The transformer delivers rated current at 0.90 power factor lagging.
2. A sustained three-phase short takes place at the 480 V terminals of the transformer.
3. The transformer delivers rated current at 0.9 Pf leading. Assume the short-circuit MVA of the source to be 500.

b. Explain why "a PT must not be energized with its secondaries short-circuited, and a CT must not be energized with its secondaries open-circuited."

2-15 Two manufacturers quoted the following on a request for the purchase of one (1) 1000 kVA, three-phase, 4160–480 V transformer:

Manufacturer	*Cost*	*Core Loss in kW*	*Copper Loss in kW*
A	$30,000	3	12
B	$35,000	2.5	9.5

Under the following operating conditions, estimate the present worth of each manufacturer's transformer over a five-year operating period.

a. Continuous operation at 75% of full-load.

b. Continuous operation at 50% of full-load.

c. Operation at full-load, 50% of the time.

Assume that the interest rate is 10%. The cost of energy is 6¢/kWh, and its annual increase is 8%.

2-16 The current (ι) drawn from a power distribution transformer is given by

$$\iota = 100 \sin \omega t + 30 \sin 3\,\omega t + 15 \sin 5\,\omega t + 10 \sin 7\,\omega t \ldots$$

Determine the required k-type of transformer.